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Synthesis Methods and Variational Methods
in Nuclear Reactor Theory, Physics, and
Computational Methods: A Bibliography

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Synthesis Methods and Variational Methods
in Nuclear Reactor Theory, Physics, and
Computational Methods: A Bibliography

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3 9338 00377 9948

Compiled by

Jean Furnish



SYNTHESIS METHODS AND VARIATIONAL METHODS IN
NUCLEAR REACTOR THEORY, PHYSICS, AND
COMPUTATIONAL METHODS: A BIBLIOGRAPHY

A search of Nuclear Science Abstracts, v. 16 (1962)-v. 23 (1969) was made for information on synthesis and variational methods in nuclear reactor theory and physics, and the computational methods involved. Abstracts are reproduced from Nuclear Science Abstracts with those from the most recent volume given first. Within each volume the arrangement is numerical.

A rough delineation of subjects has been indicated as follows:

- A. Neutron transport theory
- B. Static group diffusion theory
- C. Reactor kinetics
- D. Reactor burnup and fuel cycle analysis
- E. Reactor space-energy synthesis
- F. Reactors - general
- G. Reactor multi-dimensional space synthesis
- H. Reactor shielding

Entries not so marked are for miscellaneous material.

An index of personal authors appears at the end of the compilation.

The reader is directed also to an annotated bibliography on this subject titled "Variational Methods in Nuclear Reactor Theory" by Teodor Rosescu prepared in September 1966 at the Institute of Atomic Physics, Bucharest. The number of this document is IFA-FR-52.

Volume 23 (1969)

C

2118 NUMERICAL STUDIES OF COMBINED SPACE-TIME SYNTHESIS. Yasinsky, J. B. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 34: 158-68(Nov. 1968).

The ability to obtain accurate solutions to time-dependent group diffusion problems by simultaneously synthesizing in both the z and t dimensions is demonstrated numerically. The potential of the combined space-time synthesis method becomes apparent from several comparisons of synthesis solutions with exact (in a finite difference sense) two-group, two-dimensional, time-dependent diffusion solutions for two different reactor geometries. 16 references. (auth)

B, E

3913 SPECTRUM SYNTHESIS WITH SPATIALLY DISCONTINUOUS BASIS SPECTRA. Vaughan, E. U.; Rose, P. F.; Hausknecht, D. F. (Atoms International, Canoga Park, Calif.). Trans. Amer. Nucl. Soc., 11: 528-9(Nov. 1968).

From International Conference on the Constructive Uses of Atomic Energy, Washington, D. C. See CONF-681101.

A

4430 METHODS OF RESOLVING INTERFACE AMBIGUITIES IN DISCONTINUOUS VARIATIONAL METHODS. Becker, Martin (Rensselaer Polytechnic Inst., Troy, N. Y.). Trans. Amer. Nucl. Soc., 11: 529(Nov. 1968).

From International Conference on the Constructive Uses of Atomic Energy, Washington, D. C. See CONF-681101.

A

4431 VARIATIONAL DERIVATION OF DISCRETE-ORDINATE-LIKE APPROXIMATIONS. Natelson, N.; Gelbard, E. M. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 11: 530-1(Nov. 1968).

From International Conference on the Constructive Uses of Atomic Energy, Washington, D. C. See CONF-681101.

C

4440 SPACE-DEPENDENT DYNAMIC BEHAVIOR OF FAST REACTORS USING THE TIME-DISCONTINUOUS SYNTHESIS METHOD. Kessler, G. (Gesellschaft fuer Kernforschung mbH, Karlsruhe, Ger.). Trans. Amer. Nucl. Soc., 11: 569(Nov. 1968).

From International Conference on the Constructive Uses of Atomic Energy, Washington, D. C. See CONF-681101.

B

4442 ON THE USE OF DIFFERENT RADIAL TRIAL FUNCTIONS IN DIFFERENT AXIAL ZONES OF A NEUTRON FLUX SYNTHESIS COMPUTATION. Wachspress, E. L. (Knolls Atomic Power Lab., Schenectady, N. Y.). Nucl. Sci. Eng., 34: 342-3(Dec. 1968).

Use of radial trial functions in different axial zones to avoid flux discontinuities is discussed. Combining coefficients are defined for the various axial zones; axial neutron leakage is defined in terms of matrix notation. (M.L.S.)

C

4443 ASYMMETRIC DISCONTINUITIES IN SYNTHESIS TECHNIQUES FOR INITIAL-VALUE PROBLEMS. Becker, Martin (Rensselaer Polytechnic Inst., Troy, N. Y.). Nucl. Sci. Eng., 34: 343-4(Dec. 1968).

The bases for use of asymmetric discontinuities at overdetermined interfaces are discussed. Functional notation is used to remove symmetry considerations. The expressions for neutron flux and adjoints are shown to possess properties of asymmetric discontinuities; trial function expansions are determined for treatment of these. (M.L.S.)

F

4445 THE CALCULATION OF NEUTRON FLUX RATIOS IN CRITICAL SYSTEMS BY THE INDIRECT VARIATIONAL METHOD. Pomraning, G. C. (Los Alamos Scientific Lab., N. Mex.). Nucl. Sci. Eng., 34: 308-12(Dec. 1968).

A discussion is given of the use of the indirect variational method for generating the trial functions needed to compute a variational estimate of a homogeneous functional of the solution to an eigenvalue equation. It is shown that one use of the method leads to no difficulties, whereas another use gives meaningless results. In this latter instance, the method of weighted residuals can be used to generate the necessary trial functions. With the trial functions known, the variational estimate of the functional of interest follows by quadrature. (auth)

C

7412 (BNL-12911) ITERATIVE SOLUTIONS TO REACTOR EQUATIONS. Goldstein, Rubin (Brookhaven National Lab., Upton, N. Y.). Mar. 1967. 10p. (CONF-670501-15). Dep. CFSTI.

From International Conference on Research Reactor Utilization and Reactor Mathematics, Mexico City, Mex.

Recursion formulas for obtaining iterative solutions to reactor kinetics equations are derived. Comparisons of iterative and variational techniques for kinetics equations solution are discussed. Application of the iterative process to the solution of non-self-adjoint equations of the neutron resonance absorption-type is described. (M.L.S.)

D

11337 ANALYTICAL APPROACH TO CONTINUOUS REACTOR REFUELING. Yasukawa, S. (Japan Atomic Energy Research Inst., Tokyo). Nucl. Sci. Eng., 35: 1-13(Jan. 1969).

The analysis of the three-dimensional continuous refueling is approximated by the variational method. The axial flux distribution is treated by the trial function and expressed by the analytic expression, using the elliptic function. Introducing the perturbation technique in the flux expansion, the higher order cross-section expansion correction of the axial flux distribution is achieved by using the elliptic function as the base function in the flux expansion. It is shown that the group constants, averaged by the flux and its square, can be expressed by the simple rational function. As a special example, the nature of the eigenvalue under continuous unidirectional refueling is shown, and it is pointed out that some caution is required if the fuel burnup is evaluated by the reactivity-area method. (auth)

D
11348 ON THE USE OF DYNAMIC COORDINATE FUNCTIONS IN BURN-UP-SYNTHESIS-CALCULATIONS. Nathschlaeger, P.; Wagner, J. (Institut fuer Reaktortechnik, Seibersdorf, Austria). Atomkernenergie, 14: 5-7 (Jan.-Feb, 1969).

A dynamic coordinate method is used to optimize computer time for burnup synthesis calculations. The Newton-Cotes quadrature formulae are used to compute the matrix elements in the burnup synthesis variational codes; eigenvalues are determined by the vector iteration method. Error functions are given for assessing the accuracy of the calculations. (M.L.S.)

E
13405 (AI-AEC-12721, pp 69-81) NUCLEAR SAFETY. Vaughan, E. U. (Atomics International, Canoga Park, Calif.).

Improvements in present state kinetics codes for use in fast reactor safety calculations are described. The spectrum synthesis method in the one-dimensional static case is explored and tested by comparison with multigroup diffusion theory. Calculations are made with the XMAS-RAUM-PLEASER system on the two-region fast critical assembly ZPR-III-48. (D.C.C.)

H
13474 (AERE-R-5773 (Vol.4), pp 1098-1123) OPTIMIZATION OF NUCLEAR REACTOR SHIELDING CHARACTERISTICS ON THE BASIS OF VARIATIONAL METHODS. Abagyan, A. A.; Dubinin, A. A.; Kazanskii, Yu. A.; Orlov, V. V.; Petrov, E. E.; Pupko, V. Ya. (Gosudarstvennyi Komitet po Ispol'zovaniyu Atomnoi Energii SSSR, Moscow. Fiziko-Energeticheskii Institut).

Methods of optimizing shield composition and shape are considered. The optimization of shield composition is carried out by means of adjoint functions and perturbation theory formulas. On the basis of perturbation theory formulas the concept of a shield material efficiency function with respect to different shielding characteristics is formulated. Shielding material efficiency functions reveal the degree of sensitivity of various shielding characteristics to the variation of shield composition. Experimentally measured efficiency functions for some shielding materials are presented and are compared with calculations. On the basis of the concept of shield material efficiency functions the criteria of optimum shield composition for different cases in classical and non-classical approximations of variational calculus are given. Shield shape optimization is carried out on the basis of the Euler equation. The attenuation of radiation in shielding layers is described by the beam analysis method. The equation obtained for the function describing the form of the shielding is solved by the method of successive approximations. (22 references) (auth)

C
13576 (RPI-328-142, pp 132-51) REACTOR PHYSICS: THEORETICAL. (Rensselaer Polytechnic Inst., Troy, N. Y.).

The problem of re-entrant hole effects on time-of-flight experiments was studied; good correlation between theoretical predictions and experimental results for thermal spectrum experiments was obtained. Single-region and two-region fast

spectrum experiments were also analyzed. As an outgrowth of this work, a systematic comparison was made of the commonly used definitions of average scattering angles; it was concluded that significantly different values of the angle can be obtained. Analytic and numerical treatments of the time-dependent flux in pulsed fast systems were studied. Analytic expressions were obtained for fast reactor flux and importance spectra in terms of simple cross-section ratio correlations. The application of Green's function modes to the analysis of space-time kinetics showed that, at least in a series of test cases, spatially discontinuous Green's function modes are quite successful in predicting power shifts. The discontinuous variational method approach to space-energy problems was studied, with emphasis on the formulation of time-discontinuous initial-value problems. It was shown that, by using asymmetric discontinuities, the need for staggered discontinuities may be eliminated. It was also shown that use of asymmetric discontinuities is consistent with the forward bias of the time-like variable in an initial-value problem. (D.C.W.)

F
19133 EXTREMUM VARIATIONAL PRINCIPLES FOR THE MONOENERGETIC NEUTRON TRANSPORT EQUATION WITH ARBITRARY ADJOINT SOURCE. Buslik, A. J. (Bettis Atomic Power Lab., West Mifflin, Pa.). Nucl. Sci. Eng., 35: 303-18 (Mar, 1969).

A self-adjoint positive-definite variational principle is presented which leads to upper and lower bounds for $\langle S^*, \phi \rangle$, where $\langle S^*, \phi \rangle$ is an integral over position and angular direction of the product of the one-velocity neutron transport flux, ϕ , and an arbitrary adjoint source, S^* . The Euler equation of the functional is a new form of the one-velocity Boltzmann neutron transport equation in which the dependent variable is one-half the sum of ϕ and ϕ^* , where ϕ^* is the adjoint flux. When a trial function consisting of an expansion in spherical harmonics is used, one obtains as a lower bound for $\langle S^*, \phi \rangle$ the quantity $\langle US_1, \phi(P-N'; S_1) \rangle - \langle US_2, \phi(P-N''; S_2) \rangle$, where $S_1(r, \Omega) = [S(r, \Omega) + S^*(r, -\Omega)]/2$, $S_2(r, \Omega) = [S(r, \Omega) - S^*(r, -\Omega)]/2$, $\phi(P-N'; S_1)$ is an odd $P-N$ approximation to a problem with the same cross sections as the original problem, but with source S_1 ; $\phi(P-N''; S_2)$ is an even $P-N$ approximation to a problem with source S_2 , and U is the operator that takes a function $f(r, \Omega)$ into $f(r, -\Omega)$. (auth)

B, C
23998 (KAPL-P-3803) VARIATIONAL METHODS AND NEUTRON FLUX SYNTHESIS. Wachspress, E. L. (Knolls Atomic Power Lab., Schenectady, N. Y.). Jan. 13, 1969. 25p. (CONF-690401-1). Dep. CFSTI.

From Conference on the Effective Use of Computers in the Nuclear Industry, Knoxville, Tenn.

A review of synthesis techniques for calculating neutron fluxes in a reactor is given. Methods of selecting base functions for synthesis computations in the time domain are described; mathematical properties of these base functions are outlined. Methods of choosing the combining functions for definition of the functional are prescribed; criteria for and constraints on these choices are delineated. The concept of the trial function is explained; its role in synthesis computations is defined. The variational formalism is developed for direct and adjoint applications. Variational equa-

tions are constructed for discrete multigroup diffusion problems; error approximations are included. Application of the variational equations to spatial synthesis of the thermal spectrum problem is discussed. The method of discontinuous trial functions for multichannel synthesis formulation is detailed. 42 references. (M.L.S.)

F

23999 (WAPD-TM-771) EXTREMUM VARIATIONAL PRINCIPLES FOR THE MONOENERGETIC TRANSPORT EQUATION WITH ARBITRARY ADJOINT SOURCE (LWBR DEVELOPMENT PROGRAM). Buslik, A. J. (Bettis Atomic Power Lab., Pittsburgh, Pa.). Feb. 1969. Contract AT-11-1-GEN-14. 45p. Dep. CFSTI.

A variational method is given for determining the lower and upper bounds for the monoenergetic transport equation with arbitrary adjoint source. The functional is chosen as a form of the one-velocity Boltzmann transport equation; spherical harmonics expansions are used as trial functions. Three types of boundary conditions (b.c.) are considered: vacuum b.c., symmetric b.c., and antisymmetric b.c. Mathematical properties of the functional are deduced; stationarity criteria are applied to the functional for all three types of boundary conditions. Lower and upper bounds are obtained for the functional. A numerical example is included. 18 references. (M.L.S.)

C

24011 A GENERALIZATION OF THE FINITE DIFFERENCE APPROXIMATION TECHNIQUE APPLIED TO A NODAL MODEL OF SPACE-DEPENDENT NUCLEAR REACTOR KINETICS. Alcouffe, Raymond Edmond. Seattle, Univ. of Washington, 1968. 162p.

Thesis.

A nodal approximation of the time-dependent, one dimensional neutron diffusion equation including the effects of delayed neutrons and simple temperature feedback, has been developed. This approximation is based on a generalization of the conventional finite difference technique in that one is able to supply information of this technique about the expected solution in addition to a boundary and initial condition. The approximation is derived from a variational principle and is, therefore, consistent with the previously derived synthesis method and may be viewed as a subclass of such a method. The attributes of the particular method developed are described. (Dissert. Abstr.)

F

24109 (FEI-142) REACTIVITY INTEGRALS AND THEIR USE FOR THE SOLUTION OF CERTAIN PROBLEMS IN REACTOR PHYSICS. Stumbur, E. A. (Gosudarstvennyi Komitet po Ispol'zovaniyu Atomnoi Energii SSSR, Obninsk. Fiziko-Energeticheskii Institut). 1968. 51p. (In Russian). Dep.

The work sets forth the concept of integrals involving reactivity, specific functionals of solutions to the Boltzmann kinetic equation, which are simply connected with the k_{eff} of an arbitrary geometry reactor. Invariant integral relations are established for the Boltzmann equation in the case of critical and noncritical systems. The "method of compensated perturbations," which results in a variation equation for the integrals involving reactivity is introduced. The developed theory is used to solve some problems in reactor physics having to do with the interdependence of the critical size and composition of a reactor or the reflector size, etc. Some contrasts of theoretical results and experimental data are discussed. The work considers further possibilities for de-

veloping the methods here set forth in order to solve other problems in reactor physics. (tr-auth) (JPRS)

B

28611 AN ANOMALY ARISING IN THE COLLAPSED-GROUP FLUX SYNTHESIS APPROXIMATION. Adams, C. H.; Stacey, W. M., Jr. (General Electric Co., Schenectady, N. Y.). Nucl. Sci. Eng., 36: 444-7(June 1969).

An anomaly encountered in a flux-weighted collapsed-group synthesis problem is described. The difference between group dependent and collapsed-group synthesis is defined. In the collapsed calculations the expansion functions are used in the dual role of weighting function shapes. The reactor model to which this procedure is applied is shown; the group cross sections, used in the calculations, are defined. Results of numerical evaluation are compared, graphically, for group-dependent and collapsed-group calculations; this comparison shows the anomaly. Causes for the anomaly are analyzed. (11 references) (M.L.S.)

A

28617 SPACE-ANGLE SYNTHESIS. Swibel, Harry S. (Battelle-Northwest, Richland, Wash.); Bowes, Bradley. Nucl. Sci. Eng., 36: 435-8(June 1969).

Application of the space-synthesis critical slab and subcritical fixed source problems is described. Spatial functions are chosen for variational synthesis solution to the Boltzmann transport equation and the angular functions are determined. Results of numerical evaluation of the Boltzmann equation solutions are tabulated. (M.L.S.)

C

33104 METASTATIC METHOD IN NUCLEAR REACTOR CORE KINETICS CALCULATIONS. Galati, A. (CNEN, Rome). Nucl. Sci. Eng., 37: 30-40(July 1969).

A quasi-static method is proposed for evaluating spatial effects on nuclear reactor kinetics. The neutron flux shape is calculated approximately as an asymptotic solution of the two-group space-time diffusion equations, where delayed neutron behavior is included. Two iterative procedures are alternatively used according to the amount of reactivity involved. The first one operates until prompt criticality is reached. The second procedure replaces the first one as soon as the reactor goes superpromptcritical. The main feature of the approach adopted is the possibility of selecting an initial guess such that convergence is reached at the first iteration. The matter is then reduced to solving two eigenvalue problems. Theoretical and numerical comparisons with Henry's adiabatic model outline the main role of perturbed adjoint fluxes and correct neutron-flux shape (the second agent only for superpromptcritical excursions) in defining the generation time and reactivity. When compared with the exact solution, results of sample problems show substantial accuracy in the flux shape and amplitude. In subpromptcritical excursions, only the synthesis method is as accurate as the metastatic one and yields errors of few percent at the flux peak. In the reactivity range above prompt critical, differences between the exact results and the metastatic ones are unessential. 9 references. (auth)

B

34932 (KAPL-P-3811) ANOMALY ARISING IN THE COLLAPSED-GROUP FLUX SYNTHESIS APPROXIMATION. Adams, C. H.; Stacey, W. M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.). June 17, 1969. Contract W-31-109-Eng-52. 10p. (CONF-690609 12). Dep. CFSTI.

From 15th Annual Meeting of the American Nuclear Society, Seattle, Wash.

An anomaly arising from a flux-weighted collapsed-group synthesis calculation for an annularly-rodged cylindrical core is discussed. Results from using expansion functions as weighting shapes are described; these results are compared with actual values. Behavior of the calculated fluxes is analyzed; reasons for the behavior of the flux anomaly are explained. 11 references. (M.L.S.)

C

34942 SOLUTION OF THE SPACE-TIME DEPENDENT NEUTRON KINETICS EQUATIONS FOR A REFLECTED SLAB REACTOR. McFadden, James Hugh. Ames, Iowa, Iowa State Univ., 1968. 100p. Thesis.

A modal analysis technique is used to solve the space-time and space-frequency dependent neutron diffusion equations for a reflected slab reactor. A two-energy-group model is employed and delayed neutron effects are neglected in the solutions of the time dependent equations. The neutron flux is expanded in a series of known space functions multiplied by the corresponding time coefficients. The space functions are a form of the Green's function modes and the Kantorovich variational method is used to determine the time coefficients. A coupled mode method is applied to two time dependent problems. Approximate solutions are determined by various sets of trial functions and are compared with an exact solution as determined by a finite difference code. The frequency response of the reactor to a localized driving function also is investigated. Solutions are obtained with four sets of trial functions, of which two sets include coupled modes, and are compared at various detector positions in the fuel and reflector regions. Guides are given for the selection of trial functions based on the locations in the reactor where the frequency response is to be determined. (TSS)

B

37716 A VARIATIONAL ITERATIVE METHOD FOR THE SOLUTION OF THE ENERGY-DEPENDENT DIFFUSION EQUATION. Buslik, A. J. (Westinghouse Electric Corp., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 12: 152-3 (June 1969).

From 15th Annual Meeting of the American Nuclear Society, Seattle, Wash. See CONF-690609.

B

37717 ANOMALY ARISING IN THE COLLAPSED-GROUP FLUX SYNTHESIS APPROXIMATION. Adams, C. H.; Stacey, W. M. Jr. (General Electric Co., Schenectady, N. Y.). Trans. Amer. Nucl. Soc., 12: 151-2 (June 1969).

From 15th Annual Meeting of the American Nuclear Society, Seattle, Wash. See CONF-690609.

B

37718 ANOMALIES IN VARIATIONAL FLUX SYNTHESIS METHODS. Froehlich, Reimar (Gulf General Atomic Inc., San Diego, Calif.). Trans. Amer. Nucl. Soc., 12: 150-1 (June 1969).

From 15th Annual Meeting of the American Nuclear Society, Seattle, Wash. See CONF-690609.

E

37726 REACTION RATES AS WEIGHTING FUNCTIONS IN FAST REACTOR SPACE-ENERGY SYNTHESIS. Neuhold, R. J. (Babcock and Wilcox Co., Lynchburg, Va.); Ott, K. O. Trans. Amer. Nucl. Soc., 12: 214-15 (June 1969).

From 15th Annual Meeting of the American Nuclear Society, Seattle, Wash. See CONF-690609.

A

37732 GENERALIZED NODAL APPROXIMATION TO THE NEUTRON TRANSPORT EQUATION. Stenstrom, D. G.; Robkin, M. A. (Univ. of Washington, Seattle). Trans. Amer. Nucl. Soc., 12: 216-17 (June 1969).

From 15th Annual Meeting of the American Nuclear Society, Seattle, Wash. See CONF-690609.

A

37742 ANALYSIS OF FUEL-BLOCK WORTH MEASUREMENTS FOR A SMALL FAST ASSEMBLY IN A THREE-DIMENSIONAL CONFIGURATION. Sargis, D. A.; Cohen, S. C.; Moore, R. A. (Gulf General Atomic Inc., San Diego, Calif.). Nucl. Sci. Eng., 37: 262-70 (Aug. 1969).

A number of fuel-block reactivity-worth measurements were performed in Core No. 1 of the thermionic critical experiment. The assembly is bare and neutronicly homogeneous, but the geometry is essentially three-dimensional and the dimensions are small. A synthetic transport perturbation method is introduced for the analysis of the fuel-block worths. The agreement between experiment and analysis based upon this method is good. A useful extension of the method would be a relaxation of the first-order perturbation restriction. 16 references. (auth)

C

40424 (CONF-690401-, pp 3-15) REVIEW OF COMPUTATIONAL METHODS FOR SPACE-TIME KINETICS. Henry, A. F. (Massachusetts Inst. of Tech., Cambridge).

Approximation techniques for reactor problems are discussed with reference to computer processing. The few group diffusion theory model is outlined. Methods used in solving the space-time equations are described briefly; these include: finite differencing, time synthesis, and space-time synthesis. 22 references. (M.L.S.)

Volume 23 (1969)

F

43205 FURTHER REMARKS ON THE CALCULATION OF RATIOS IN CRITICAL SYSTEMS. Pomraning, G. C. (Gulf General Atomic Inc., San Diego, Calif.). J. Nucl. Energy, 22: 631(Oct. 1968).

A comparison is made of the two variational principles of Lewins [NSA 20: 18053] and Pomraning [NSA 21: 32025] for calculating the ratio of reaction rates in a critical system. The simplicity of the principles from a computational point of view is considered. (UK)

B, G

45128 (WAPD-TM-891) WANSY: A PROGRAM TO SOLVE THE STATIC TWO-DIMENSIONAL GROUP DIFFUSION EQUATIONS BY SYNTHESIS METHODS. Yasinsky, J. B. (Bettis Atomic Power Lab., Pittsburgh, Pa.). July 1969. Contract AT-11-1-GEN-14. 54p. Dep. CFSTI.

WANSY is a synthesis program that combines one-dimensional trial functions obtained from normal, adjoint or direct input WANDA solutions via axially dependent mixing coefficients to yield two-dimensional flux shapes. The trial functions may be axially discontinuous (different sets of trial functions may be used in different axial regions), and different numbers of trial functions may be used in different axial regions. The synthesis approximation may be group-dependent, partially or totally group-collapsed, and weight function scaling factors may be input to improve collapsed group synthesis solutions. The synthesis approximations and required input are described. 13 references. (auth)

E, A

45129 SPACE-ENERGY FLUX SYNTHESIS IN TRANSPORT THEORY. Lancefield, M. J. (Brookhaven National Lab., Upton, N. Y.). Nucl. Sci. Eng., 37: 423-42(Sept. 1969).

The efficacy of the overlapping group method in fast-reactor analysis is investigated and tested on an idealized fast-reactor configuration. A full transport-theory treatment is adopted and the overlapping group equations are derived by the indirect use of a variational principle. A number of refinements to the basic method have been examined and serve to demonstrate that with a judicious choice of variational functional and trial functions it is possible to obtain accurate estimates not only of the reactivity and other integral quantities but also of the detailed flux. These include: leaving both the space/angle and energy dependence of the trial functions to be determined by the variational principle, incorporating discontinuous trial functions, and the use of a new variational principle for criticality problems that leads to estimates of homogeneous functionals of the unknown flux. 24 references. (auth)

A

47295 (CNM-R-2(Vol.1), pp. 238-49) VARIATIONAL DEVELOPMENT OF S_N THEORY. Schreiner, Sheldon; Selengut, D. S. (California Univ., Berkeley).

An alternative derivation of the S_N equations for the one velocity transport problem in plane geometry is described by using a variational approach. This permits retaining the angular segmentation picture of the neutron angular distribution, but leads to an unambiguous result for the entire set of equations as well as the boundary and continuity conditions to be imposed at interfaces. The required variational formalism is outlined. The direct and adjoint angular flux distributions implied by the angular seg-

mentation approximation are used as trial functions, leading to a set of differential equations and boundary conditions which can be interpreted as the analogs of either the Mark or Marshak procedures used in the spherical harmonics approximation. 4 references. (auth)

F

47296 (CNM-R-2(Vol.1), pp. 250-66) GENERALIZED VARIATIONAL PRINCIPLES FOR REACTOR ANALYSIS. Pomraning, G. C. (Brookhaven National Lab., Upton, N. Y.).

Variational principles are compared for derivation and accuracy. The basic ingredient in the derivations is a particular use of Lagrange multipliers. Using the technique of postulating the principles and then proving the accuracy, it is shown that more general variational principles can be established. 10 references. (auth)

C

47302 (CNM-R-2(Vol.2), pp. 994-1014) A COMPARISON OF SOME NUMERICAL METHODS FOR SOLVING THE EQUATIONS OF REACTOR DYNAMICS. Szeligowski, John; Hetrick, David (Arizona Univ., Tucson).

The numerical solution of the equations of reactor dynamics is frequently complicated by the presence of one response time that is very short compared to the overall time-response of the system (e.g., slow transients in a fast reactor). The number of time steps in a standard method (e.g., Adams, Runge-Kutta) may become prohibitively large. Some computational methods that circumvent this limitation are reviewed and compared. The methods fall into two general classes: those based on expansions in a small parameter, and those using integral equations in which the slowly varying factors in the integrands are approximated by known functional forms. The former, which include the "prompt-jump approximation," may be classified as singular perturbation expansions; these are typically restricted to a limited range of some parameter and hence will fail for a non-stationary system whenever some time-dependent coefficient strays outside a prescribed limit (e.g., a slow ramp reactivity input approaching prompt critical). Integral methods do not have this latter limitation; the methods treated may be classified as iterative, variational, and largest-eigenvalue methods. Some typical examples of slow transients in fast reactors demonstrate reduction of the number of time steps by a factor of the order of 10^4 . 19 references. (auth)

B, C

49651 (ANL-7410, pp. 387-442) REACTOR COMPUTATION METHODS AND THEORY. (Argonne National Lab., Ill.).

- REACTORS — neutron diffusion equations for two-dimensional cylindrical, use of discontinuous variational synthesis technique for solution static multigroup
- kinetic equations for, formulations of microscopic multigroup cross sections for solution to
 - neutron diffusion calculations for, use of group-space-dependent bucklings to simulate coordinate system transformations for
 - kinetic equations for, error analyses of linked-difference,

REACTORS, WATER COOLED—neutron diffusion equations for two-dimensional cylindrical use of discontinuous variational synthesis technique for solution static multigroup —kinetic equations for, formulations of microscopic multigroup cross sections for solution to (M.L.S.)

D
49654 (GA-9423) SCANAL: A SINGLE CHANNEL SYNTHESIS DEPLETION CODE WITH TRIANGULAR MESH IN THE HORIZONTAL PLANE. Traylor, R. C.; Malakhof, V.; Leighton, S. C. (Gulf General Atomic, Inc., San Diego, Calif.). July 1, 1969. Contract AT(04-3)-633. 143p. Dep. CFSTI.

The digital computer code SCANAL is described. SCANAL is designed to synthesize a three-dimensional power distribution from GAUGE and RELOAD FEVER analyses and to utilize the COPE code to calculate complete distributions of core moderator fuel, and coolant temperatures. The program is written for the UNIVAC 1108. (6 references) (D.C.C.)

G
52103 (CEA-R-3713) SPACE SYNTHESIS: AN APPLICATION OF SYNTHESIS METHOD TO TWO AND THREE DIMENSIONAL MULTIGROUP NEUTRON DIFFUSION EQUATIONS. Nguyen-Ngoc, Hoan (Commissariat a l'Energie Atomique, Saclay (France). Centre d'Etudes Nucleaires). May 1969. 68p. (In French). Dep. CFSTI (U. S. Sales Only).

In order to reduce computing time, two- and three-dimensional multigroup neutron diffusion equations in cylindrical, rectangular (X, Y), (X, Y, Z) and hexagonal geometries are solved by the method of synthesis using an appropriate variational principle (stationary principle). The basic idea is to reduce the number of independent variables by constructing two or three-dimensional solutions from solutions of fewer variables, hence the name "synthesis method." Whatever the geometry, one is led to solve a system of ordinary differential equations with matrix coefficients to which one can apply well-known numerical methods: Chebyshev's polynomial method, Gaussian elimination. Numerical results furnished by synthesis programs written for the IBM 7094, the IBM 360-75, and the CDC 6600 computers, are confronted with those which are given by programs employing the classical finite difference method. (auth)

E
52443 (AI-AEC-12820) SPECTRUM SYNTHESIS IN FAST REACTOR ANALYSIS. Vaughan, E. U.; Rose, P. F.; Hausknecht, D. F. (Atomics International, Canoga Park, Calif.). June 30, 1969. Contract AT(04-3)-701. 106p. Dep. CFSTI.

The synthesis method has been applied to fast reactor calculations in the form of spectrum synthesis, in which the neutron distribution in a region is represented approximately as a linear combination of a few predetermined energy spectra (the basis spectra), the coefficients being space-dependent functions (the mixing functions). Equations to determine the mixing functions are obtained in several ways which include the standard variational and Galerkin methods, and the group balance method in which the original multigroup equations are replaced by conditions of neutron balance in a number of coarse groups equal to the number of basis spectra. The equations have been cast into a form suitable for solution by the one-dimensional, few-group,

diffusion-theory code RAUM, which has been generalized by relaxing the restrictions that the matrix of diffusion coefficients be diagonal and the scattering transfer matrix be triangular. Codes have been written to translate multigroup cross-section libraries into the matrices required by RAUM, and also to construct synthesized spectra from the RAUM output. This can be done even when different regions use different basis spectra, provided the group balance method is used, with the same coarse groups in all regions. A number of calculations using this synthesis scheme are reported, mostly on the critical assembly ZPR-3-48, with or without partial voiding of sodium, using a two-fold or three-fold spectral basis consisting of typical spectra, found in a calculation with the one-dimensional multigroup diffusion-theory code CAESAR. Comparison of the RAUM and CAESAR calculations indicates that the multiplication factors generally agree to a few tenths of a percent, and that power distributions, and spectra at individual points (especially in the core), usually agree well also. But these favorable indications are qualified by occasional failures, in which RAUM gives very unphysical results. Such failures can be avoided by proper choice of the coarse groups for the group balance method, and evidence is presented that had choices can be recognized by studying the RAUM input matrices they yield, without the need for a complete RAUM calculation. (auth)

C
52462 (KFK-781/1) NUMERICAL SOLUTION OF THE POSITION-DEPENDENT DYNAMIC EQUATIONS FOR FAST BREEDER REACTORS USING A VARIATIONAL PRINCIPLE. Kessler, G. (Kernforschungszentrum, Karlsruhe (West Germany). Institut fuer Reaktorentwicklung). Aug. 1968. 123p. (In German). (EUR-3957). Dep.

The time-, position-, and energy-dependent multigroup neutron diffusion equations are formulated for fast breeder reactors; boundary conditions are specified. Heat transfer equations and boundary conditions for fuel-to-coolant heat flow are given. The feedback equations for the fuel, cladding, coolant, and structural materials are listed. Methods for solution to the position dependent kinetic equations are discussed; the Kantorovitch variational principle is described in detail. The systems of differential equations for the time function of the nonstationary multigroup neutron flux and precursors are formulated. Application of the Kantorovitch variational principle to the solutions of these equations is detailed. The calculational model used as a sample problem for illustrating the variational solution is described. Neutronics parameters are tabulated. Results of power distribution, reactivity, and neutron flux calculations are described; data are shown graphically. 61 references. (M.L.S.)

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A, B
1586 VARIATIONAL ESTIMATE OF THE DIFFUSION COEFFICIENT IN A LATTICE WITH CAVITIES. Devooght, Jacques (Univ. Libre, Brussels). *Nukleonik*, 10: 119-28 (Sept. 1967).

The theory of Benoist is modified by using a definition of the diffusion coefficient taking into account importance weighting. It is shown that in the one group model a quadratic weighting by the fine structure flux exists instead of a linear one, and that the macroscopic variation of importance across the reactor makes the absorption correction of Benoist disappear. The Boltzmann equation for the flux perturbed by an imposed gradient is thrown into a self-adjoint form by a method of Vladimirov. A variational expression of the diffusion coefficient is given. The expression found is closely related to Dirichlet principle, and involves only differential expressions. It is shown that diffusion theory trial functions in a non-capturing medium lead to the "electrostatic" approximation. By direct extension of a method of Carter, the variational expression found is shown to be identical to the one derived above. It is shown that the relative variation of the diffusion coefficient, obtained by the introduction of cavities, is equal to the relative variation of potential energy obtained by the introduction of a dielectric cavity in a constant electric field. Henceforth, classical expressions derived by Polya and Szegő, give an explicit value for the diffusion coefficient in the presence of large cylindrical cavities of arbitrary cross-sections. Electrical analogues are briefly discussed. (auth)

C
1882 (KAPL-P-3392) TIME SYNTHESIS: A STUDY OF SYNTHESIS MODES AND WEIGHTING FUNCTIONS. Rydin, Roger A. (Knolls Atomic Power Lab., Schenectady, N. Y.). Nov. 7, 1967. Contract W-31-109-eng-52. 25p. (CONF-671102-31). Dep. CFSTI.

From 15th Conference on Remote Systems Technology and Atom Fair, Chicago, Ill.

The Time-Synthesis Method has been used to investigate the effects of the choice of trial functions and weighting functions on the results of synthesis calculations. It was found that the time-dependent total power in a reactor could be adequately predicted using any of the methods of weighting considered (region-balance, flux, or adjoint), but that the spatial flux distribution was more difficult to synthesize. In particular, region-balance weighting did not give very good spatial results, flux weighting gave better results which were sensitive to the group spectra, and adjoint weighting gave the best results. A singularity in the total removal matrix was observed during the course of the transient in one of the adjoint weighted problems. The synthesis results were sensitive to the choice and number of trial functions used. In particular, the asymptotic flux shape, corresponding to the final state of the reactor, was not the best mode to use because of delayed neutron holdback effects. Similarly, the addition of transition modes between the initial and final states were beneficial in improving the time-dependent flux shapes calculated early in the transient. Qualitative criteria were developed for choosing synthesis modes based upon the initial eigenvalue separation of the core, the time scale of the transient relative to the delayed neutron half-lives, and the total magnitude of the perturbation. (auth)

C
3944 VARIATIONAL FUNCTIONALS FOR SPACE-TIME NEUTRONICS. Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.). *Nucl. Sci. Eng.*, 30: 448-63 (1967).

Two variational principles, which may be used as the basis of modal expansion approximations for treatment of space-time dependence of neutron balance equations for describing reactors, are given. Time-dependent multigroup P_1 equations are written in a variational matrix notation; expansion functions which satisfy spatial and temporal boundary conditions are obtained. A time-integrated approximation is then used to obtain equations in spatial notation only. 12 references. (M.L.S.)

C
5982 APPLICATION OF TIME-SYNTHESIS TECHNIQUES TO COUPLED-CORE-TYPE REACTORS. Yasinsky, J. B. (Bettis Atomic Power Lab., West Mifflin, Pa.). *Trans. Amer. Nucl. Soc.*, 10: 570-1 (Nov. 1967).

From 15th Conference on Remote Systems Technology and Atom Fair, Chicago, Ill., Nov. 5-9, 1967. See CONF-671102.

C, D
5986 APPLICATION OF VARIATIONAL SYNTHESIS TO OPTIMAL CONTROL. Stacey, Weston M. Jr. (General Electric Co., Schenectady, N. Y.). *Trans. Amer. Nucl. Soc.*, 10: 573-4 (Nov. 1967).

From 15th Conference on Remote Systems Technology and Atom Fair, Chicago, Ill., Nov. 5-9, 1967. See CONF-671102.

B, G
6012 MULTIGROUP, TWO-DIMENSIONAL SYNTHESIS CALCULATIONS FOR FAST POWER REACTORS. Hutchins, B. A.; Kelley, M. D.; Gyorey, G. L. (General Electric Co., Smyvale, Calif.). *Trans. Amer. Nucl. Soc.*, 10: 526-7 (Nov. 1967).

From 15th Conference on Remote Systems Technology and Atom Fair, Chicago, Ill., Nov. 5-9, 1967. See CONF-671102.

C
6146 TIME SYNTHESIS: A STUDY OF SYNTHESIS MODES AND WEIGHTING FUNCTIONS. Rydin, R. A. (General Electric Co., Schenectady, N. Y.). *Trans. Amer. Nucl. Soc.*, 10: 569-70 (Nov. 1967).

From 15th Conference on Remote Systems Technology and Atom Fair, Chicago, Ill., Nov. 5-9, 1967. See CONF-671102.

F, C
6157 SEMIDIRECT LEAST-SQUARES VARIATIONAL METHODS AND INITIAL-VALUE PROBLEMS. Becker, Martin (Rensselaer Polytechnic Inst., Troy, N. Y.). *Trans. Amer. Nucl. Soc.*, 10: 550-1 (Nov. 1967).

From 15th Conference on Remote Systems Technology and Atom Fair, Chicago, Ill., Nov. 5-9, 1967. See CONF-671102.

F

7626 VARIATIONAL PRINCIPLE OF THE NEUTRON TRANSPORT THEORY AND PSEUDOENTROPY PRODUCTION. Helms, G. (Institut fuer Magnetohydrodynamik, Jena, Ger.). Monatsber. Deut. Akad. Wiss., Berlin, 9: 186-91(1967). (In German).

The relation between the variational principle of reactor theory and the principle of the minimum pseudoentropy production was derived by assuming a very simple model of the processes in the reactor. Usually, however, there is a qualitative relation between the two principles according to the usual model. (J.S.R.)

A

12139 (BNWL-472, pp 2,1-3) REACTOR THEORY AND CODE DEVELOPMENT. (Battelle-Northwest, Richland, Wash. Pacific Northwest Lab.).

Eighteen isotopes for which additional and more accurate data have been added to the BNW Master Cross Section Library are listed. An angular synthesis approximation method applied to analyzing a thin critical slab is outlined, and eigenvalues and total fluxes are presented for several different thicknesses. (H.D.R.)

C

14296 (RPI-328-100, pp 140-87) REACTOR PHYSICS: THEORETICAL. (Rensselaer Polytechnic Inst., Troy, N. Y.).

Using nuclear data intended for fast reactor large oxide cores, the inverse moderating ratio and the fission-to-absorption ratio were correlated and the adjoint spectrum was evaluated. A preliminary analysis of a large carbide core was also made using the same data. The cross section correlations and the comparison of analytic and multigroup adjoint spectra are shown for the carbide system. The possibility of using semidirect least-squares variational methods for initial-value problems in fast reactor analysis is examined. Static tilt test problems using Green's function modes and variational methods were performed for test reactors consisting of bare slabs of varying thicknesses containing monoenergetic neutrons. Directional and scalar fluxes as a function of hole depth are presented in an examination of re-entrant hole perturbation effects in water. The time dependent equation for the slowing down of a fast neutron pulse in a finite hydrogenous medium was studied. The problems of Doppler broadening of p-wave and s-wave neutron resonances were formulated and examined. A theoretical study of time moments for thermal neutrons was initiated using a single relaxation time model. Results obtained using a model for the analysis of time dependent neutron transport are presented. The accuracy of the importance-shape method for determining the geometric buckling in reflected reactors is investigated. (H.D.R.)

A

14305 STRATEGY FOR THE APPLICATION OF SPACE-ANGLE SYNTHESIS TO PRACTICAL PROBLEMS IN NEUTRON TRANSPORT. Natelson, M. (Bettis Atomic Power Lab., West Mifflin, Pa.). Contract AT(11-1)-gen-14. Nucl. Sci. Eng., 31: 325-36(1968). (WAPD-T-1986).

A strategy is proposed for the application of space-angle synthesis (SAS) to the finding of solutions for practical nuclear reactor neutron transport problems. A simple SAS approximation is derived. Trial functions for the approximations are to be created for each mesh point used in describing a set of similar problems

which are to be solved. The strategy is concerned with constructing problems that are simpler than, but representative of, the set of problems finally to be solved. It is from transport solutions of these representative problems that the SAS trial functions are to be formed. This strategy and the simple SAS approximation are applied successfully to several sets of similar problems for which diffusion theory is inadequate. (auth)

C

14309 ANALYSIS OF PULSED FAST-NEUTRON SPECTRA IN MULTIPLYING ASSEMBLIES. Jenkins, J. D.; Daltch, P. E. (Rensselaer Polytechnic Inst., Troy, N. Y.). Nucl. Sci. Eng., 31: 222-33(1968).

A simple model is developed to describe the time-dependent neutron spectrum in pulsed systems whose decay may be dominated by either a fundamental or a pseudofundamental mode. Such systems include a large class of fast multiplying assemblies and thermal nonmultiplying assemblies. The simple mode provides qualitative understanding of the role played by the fundamental or pseudofundamental mode in the kinetic evolution of the time-dependent neutron flux and, when optimized by a variational principle, gives excellent quantitative descriptions of the flux for a wide range of systems. Trial functions are presented which, when adjusted with a suitable variational principle, provide a good estimate of the shape and decay rate of the dominant reproducing mode of such systems. The method works well for systems where a fundamental mode exists and is also applicable in the range where pseudofundamental mode behavior is observed. Eigenfunction eigenvalue solutions are obtained for the fast multiplying system GODIVA and these, together with similar solutions for beryllium, provide a basis of comparison for the variational methods. The investigation shows that care should be exercised in associating reactivity and period parameters with subcritical systems because the flux shape is changing substantially and the major regenerative mode is not isolated when the eigenvalue associated with this mode lies in the continuum. In the farther subcritical region an example shows a complete lack of a single dominant mode. 19 references. (auth)

F

14310 ON THE PERTURBATION FORMULA FOR RATIOS IN CRITICAL SYSTEMS. Ribaric, M. (Inst. Josef Stefan, Ljubljana, Yugoslavia). J. Nucl. Energy, 21: 899-902(Dec. 1967).

Usachev (1964) derived by physical reasoning a perturbation formula for computing the changes in the frequency ratios of various processes in a critical reactor caused by a change in its composition. Lewins (1966) and Pomraning (1967) constructed analogous variational principles for this purpose. Fischer (1967) inferred an equivalent perturbation formula from the asymptotic behaviour of the perturbed critical reactor. Usachev's perturbation formula is derived mathematically via Brillouin-Wigner perturbation formula of quantum mechanics. (auth)

B, E

14318 FLUX SYNTHESIS USING GREEN'S FUNCTION IN TWO-DIMENSIONAL GROUP DIFFUSION EQUATIONS. Kobayashi, Keisuke (Kyoto Univ.). Nucl. Sci. Eng., 31: 91-101 (1968).

The group diffusion equations in two dimensions are solved by assuming the separation of variables sectionally. Using one-dimensional Green's functions, the two-dimensional diffusion equations are transformed into two sets of one-dimensional three-point difference equations at fine-mesh points. Assuming that the separation of variables of x and y coordinates is possible in a coarse mesh in a reactor, the two sets of one-dimensional difference equations are solved by the alternating direction iteration method. Sample calculations for ^{235}U - H_2O thermal reactors show that this method gives fairly good results with few coarse and fine meshes and the computation time can be considerably reduced compared with the usual finite difference method. 9 references. (auth)

F, B
14320 USE OF DUAL VARIATIONAL PRINCIPLES FOR THE ESTIMATION OF ERROR IN APPROXIMATE SOLUTIONS OF DIFFUSION PROBLEMS. Yasinsky, J. B.; Kaplan, S. (Bettis Atomic Power Lab., West Mifflin, Pa.). Nucl. Sci. Eng., 31: 80-90(1968).

A method for using reciprocal variational problems to develop figures of merit for approximate solutions of diffusion problems is presented. The theory of the reciprocal problems is described in both a continuous and discrete context. Connections with the method of Slobodyansky are discussed. A stratergem is presented for extending the method to the (non-self-adjoint) group-diffusion case. Limitations of the method are discussed and numerical examples given. It is concluded that the method is useful in one-, two-, and perhaps in small three-dimensional problems but is probably computationally not practical for full-blown, detailed, three-dimensional calculations. 9 references. (auth)

B, G
16328 APPLICATION OF SYNTHESIS TECHNIQUES TO THE CALCULATIONS OF THREE-DIMENSIONAL REACTIVITY-COEFFICIENT DISTRIBUTIONS. Bear, J. L.; Judge, F. D.; Venerus, E. R. (General Electric Co., Schenectady, N. Y.). Contract W-31-109-eng-52. Nucl. Sci. Eng., 31: 349-53(Feb. 1968). (KAPL-P-3312).

Application of synthesis techniques to calculation of three dimensional reactivity coefficients is discussed. An expression using this synthesis method is formulated. Comparisons between synthesized calculations and diffusion theory calculations are presented graphically. (M.L.S.)

B
16329 ANOMALIES ARISING FROM THE USE OF ADJOINT WEIGHTING IN A COLLAPSED GROUP-SPACE SYNTHESIS MODEL. Yasinsky, J. B.; Kaplan, S. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 31: 354-8 (Feb. 1968).

The use of the adjoint weighting procedure, as used in a collapsed group-space synthesis model, is discussed. Test problems are used to show how use of weighted adjoints destroys the utility of this collapsed-group synthesis. The use of adjoint weighting techniques is compared to use of Galerkin weighting. (M.L.S.)

F
18598 VARIATIONAL PRINCIPLES FOR CALCULATION OF ARBITRARY RATIO OF PROCESSES IN CRITICAL ASSEMBLIES. Dwivedi, S. R. (Bhabha Atomic Research Centre, Trombay, India). J. Nucl. Energy, 22: 123-5(Feb. 1968).

It is shown that the principles of Lewins and Pomraning are the same to the first order, and that Pomraning's principle can be derived from that of Lewins. (UK)

C
18606 GENERALIZED FORMULATION OF POINT NUCLEAR REACTOR KINETICS EQUATIONS. Becker, Martin (Rensselaer Polytechnic Inst., Troy, N. Y.). Contract AT(30-3)-328. Nucl. Sci. Eng., 31: 458-64(1968).

The most general current formulations of the point reactor kinetics equations permit the flux shape function to be time dependent. This permissibility has led to the development of a class of space-time analyses referred to as adiabatic or quasi-static. The use of time-independent importance weighting, however, can lead to difficulties, as is shown in an example. Point kinetics equations are derived from a variational principle in such a way as to permit time-dependent importance shape functions. "Extra" terms due to the explicit time dependence of the shape functions appear, and normalization conditions are obtained by which these terms can be eliminated. Additional differences from conventional form appear if one chooses to use different importance shape functions for flux and precursor equations, but these differences can be neglected for many cases of practical interest. (auth)

18608 SYNTHESIS METHOD OF UNCERTAINTY ANALYSIS IN NUCLEAR REACTOR THERMAL DESIGN. Fenech, Henri; Queron, Henri M. (Massachusetts Inst. of Tech., Cambridge). Nucl. Sci. Eng., 31: 505-12(1968).

The principal methods of core design uncertainty analysis are critically reviewed. The overconservatism of the Deterministic Method, which aims at ensuring that the design limits cannot be exceeded in the most loaded channel (or at the most loaded spot), leads to a probabilistic approach in which the probability of such an event is evaluated. Recent work in this direction is discussed. It is emphasized, however, that a probabilistic reliability evaluation must cover the whole core, and not only its most heavily loaded element. The Synthesis Method fulfills this requirement without demanding the use of computers. The Synthesis Method also allows the use of a realistic space-dependent reliability criterion. The various methods under review are compared in their application to a fast gas-cooled reactor core. The power levels corresponding to a given reliability are calculated and the Synthesis Method is seen to be more conservative than the classical Statistical Method and less conservative than the Deterministic Method. 16 references. (auth)

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B

22653 (ANL-7419) REACTOR DEVELOPMENT PROGRAM. Progress Report, January 1968. (Argonne National Lab., Ill.), Feb. 27, 1968. Contract W-31-109-eng-38. 158p. Dep. CFSTI.

REACTORS—excursions in, coherence effects in super prompt critical, (T); neutron flux synthesis in, variational method for, (T); core deformation under impulsive loading, analysis of cylindrical, (T)

NEUTRONS—flux synthesis in reactors, variational method for, (T)

C

25130 (KAPL-P-3425) SPACE-TIME REACTOR KINETICS DEVELOPMENT AT KAPL. Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.), Mar. 1968. Contract W-31-109-eng-52. 42p. (CONF-680414-1). Dep. CFSTI.

From Conference on Industrial Needs and Academic Research in Reactor Kinetics, Upton, N. Y.

Efforts in the development of methods for the calculation of transient power distributions in operating reactor cores during a variety of plausible transients are reviewed. The development of Xenon spatial transient analysis and space-time flux synthesis is examined. Studies on defining nodal coupling coefficients and on the influence of spatial effects on the kinetics of low source reactor startups are also discussed. (H.D.R.)

B

25140 INTERFACE CONDITIONS FOR FEW-GROUP NEUTRON DIFFUSION EQUATIONS WITH FLUX-ADJOINT WEIGHTED CONSTANTS. Buslik, A. J. (Bettis Atomic Power Lab., West Mifflin, Pa.), Contract AT(11-1)-Gen-14. Nucl. Sci. Eng., 32: 233-40(1968). (WAPD-T-2073).

Few-group diffusion equations are derived from variational principles. It is shown that by proper choice of trial function it is possible to derive a few-group theory in which interface boundary conditions of continuity of few-group fluxes and currents are obtained, even when the few-group constants are obtained by flux-adjoint weighting. The analysis is facilitated by the use of functionals that incorporate the interface condition of flux continuity by means of Lagrange multipliers. Two functionals are used to give two variants of the theory. Both functionals have as Euler equations the P-1 approximation to the time-independent, eigenvalue form of the energy-dependent transport equation. In addition, the current and flux interface boundary conditions are part of the complement of Euler conditions of the functionals. The functionals admit trial functions discontinuous in space and energy. The two functionals differ in that one has both flux and current arguments, whereas the other has only flux arguments, and yields the P-1 equations in second-order diffusion form. (auth)

A

27481 (BNWL-634, pp 2.1-8) REACTOR THEORY AND CODE DEVELOPMENT. (Battelle-Northwest, Richland, Wash. Pacific Northwest Lab.).

A space-angle synthesis approximation of the transport equation has been developed which shows improved accuracy over the P1 approximation. The problem considered is an infinite homogeneous slab with a uniform source. The Battelle Revised Thermos code has been modified by the addition of an improved current calculation routine, fission cross section averaging both microscopically per isotope and macroscopically for the cell, and the capability to handle up to eight mixtures. A computer program entitled HTH,

comprising an overlay of codes HRG, Battelle Revised Thermos, and HFN, has been coded to provide a code package for use in reactor analysis. Modifications have been made to the HAMMER code to make it more compatible with other codes in use at PNL. (auth)

C

30151 (KAPL-M-6742) STUDY OF THE MULTICHANNEL SYNTHESIS METHOD FOR SPACE-TIME NEUTRONICS. Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.), Nov. 1967. Contract W-31-109-Eng-52. 69p. Dep. CFSTI.

A multichannel space-time synthesis model for the calculation of nonseparable reactor transients is developed from a variational principle which admits expansion functions that are discontinuous in space and time. In each of many spatial regions, the flux during each interval of time is expanded in known functions of position with unknown expansion coefficients. The accuracy of the method, and its superiority with respect to the conventional single-channel space-time synthesis method, are demonstrated by several numerical examples. (auth)

C

30351 APPLICATION OF TIME-SYNTHESIS TECHNIQUES TO COUPLED CORE REACTORS. Yasinsky, J. B. (Bettis Atomic Power Lab., West Mifflin, Pa.). Nucl. Sci. Eng., 32: 425-9(1968).

The application to coupled core reactor problems of the multi-mode time synthesis approximation using trial functions $H_k(r)$ which are easily obtained is described. (D.C.C.)

F

32033 CANONICAL AND INVOLUTORY TRANSFORMATIONS OF VARIATIONAL PROBLEMS INVOLVING HIGHER DERIVATIVES. Kaplan, S. (Westinghouse Electric Corp., West Mifflin, Pa.). J. Math. Anal. Appl., 22: 45-53(Apr. 1968).

Because of their value in this type of problem it is clearly desirable to extend the notions of canonical and involutory transformation to variational problems involving higher derivatives. Although this extension is straightforward it does not seem to be readily available in the literature. Thus an exposition of the process is given here. Only problems containing second derivatives are considered; however, this case seems sufficient to indicate the pattern for problems with higher derivatives. (W.D.M.)

A

32434 SPACE-ANGLE SYNTHESIS. Zwibel, H. S. (Battelle Memorial Inst., Richland, Wash.); Bowes, B. Trans. Amer. Nucl. Soc., 11: 174(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

B, C

32800 A SYNTHESIS METHOD FOR CALCULATING TEMPERATURE, POWER, AND BOWING REACTIVITY COEFFICIENTS OF FAST POWER REACTORS. Blomberg, Pehr E. (Argonne National Lab., Idaho Falls, Idaho). Trans. Amer. Nucl. Soc., 11: 208-9(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

B

32825 NUMERICAL STUDY OF SINGLE-CHANNEL FLUX SYNTHESIS. Adams, Cyrus H.; Rydin, Roger A.; Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.). Trans. Amer. Nucl. Soc., 11: 169-70(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

E

32826 SPACE-ENERGY SYNTHESIS TECHNIQUES FOR FAST REACTOR CALCULATIONS. Murley, T. E.; Williamson, J. W. (Westinghouse Electric Corp., Madison, Pa.). Trans. Amer. Nucl. Soc., 11: 174-5(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

C

32829 VARIATIONAL MULTICHANNEL SPACE-TIME SYNTHESIS MODEL FOR THE ANALYSIS OF NONSEPARABLE TRANSIENTS. Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.). Trans. Amer. Nucl. Soc., 11: 170-1(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

C

32830 COMBINED SPACE-TIME SYNTHESIS MODEL: NUMERICAL COMPARISONS WITH EXACT TWO-GROUP TWO-DIMENSIONAL TRANSIENT SOLUTIONS. Yasinsky, J. B.; Lynn, L. L.; Kaplan, S.; Porsching, T. A. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 11: 172-3(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

A

32981 ELEMENTARY SYNTHETIC APPROACH TO THREE-DIMENSIONAL TRANSPORT PERTURBATION THEORY. Sargis, D. A.; Cohen, S. C. (Gulf General Atomic Inc., San Diego, Calif.). Trans. Amer. Nucl. Soc., 11: 173-4(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

F

33139 EXTREMUM VARIATIONAL PRINCIPLES FOR THE MONOENERGETIC TRANSPORT EQUATION WITH ARBITRARY ADJOINT SOURCE. Buzlik, A. J. (Westinghouse Electric Corp., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 11: 315(June 1968).

From 14th Annual Meeting of the American Nuclear Society, Toronto. See CONF-680601.

F

35202 COMPLEMENTARY VARIATIONAL PRINCIPLES AND THEIR APPLICATION OF NEUTRON TRANSPORT PROBLEMS. Pomraning, G. C. (General Dynamics Corp., General Atomic Div., San Diego, Calif.). J. Math. Phys. (N. Y.), 8: 2096-2108(Oct. 1967).

Several variational principles are developed which give upper and lower bounds for the linear functional (S, ϕ) , where ϕ is the solution of the inhomogeneous equation $H\phi = S$ with H a self-adjoint, positive-definite, linear operator. Some of the principles bound this functional only with respect to small or local variations, whereas others give bounds for arbitrary variations. Several of the results obtained coincide with those of other authors widely scattered throughout the literature, and we show that these principles have a common origin. Other results given are new. Examples of the use of these principles are taken from the field of neutron transport theory, and both the linear Boltzmann or transport equation and the diffusion equation are used. One interesting result is that certain "exact" values of the extrapolated endpoint for the Milne problem, which have been reported in the literature, fall, due to numerical inaccuracies, outside the bounds computed here. (auth)

B, G

39919 CALCULATION OF NONSEPARABLE REACTOR GEOMETRIES USING THE EXPANDED FLUX SYNTHESIS METHODS. Schaeffler, Hermann (Technische Hochschule, Stuttgart). Nukleonik, 11: 91-5(May 1968). (In German).

Multigroup diffusion theory is used to formulate matrix expressions for the neutron flux. These expressions are then used in the expanded flux synthesis technique to describe two- and three-dimensional special reactor configurations in rectangular coordinates having partial absorber plates. (M.L.S.)

B, C

39920 EXPANDED FLUX SYNTHESIS METHOD. Schaeffler, Hermann (Technische Hochschule, Stuttgart). Nukleonik, 11: 84-9(May 1968). (In German).

Multigroup theory is used as the basis for the kinetics equations for cylindrical reactors having discrete control rods. The effects of these control rods on the neutron flux are determined directly; an expanded flux synthesis technique is used to reduce the number of unknowns in the final system of differential equations. An example problem is given. (M.L.S.)

C

39933 APPLICATION OF VARIATIONAL SYNTHESIS TO THE OPTIMAL CONTROL OF A POINT REACTOR MODEL. Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.). Nucl. Sci. Eng., 33: 257-60(1968).

The techniques of variational synthesis are applied to obtain a solution for the optimum reactivity control of a point reactor model, with temperature feedback, subject to a general type of optimizing function. An integral equation formulation of the neutron dynamics allows the solution of a two-point boundary value problem that is frequently associated with the differential equation formulation of the optimal control problem to be replaced with a system of algebraic equations. (auth)

A

39936 VARIATIONAL APPROACH TO THE SELECTION OF THE DIRECTION SETS IN THE DISCRETE S_n APPROXIMATION TO NEUTRON TRANSPORT THEORY. Jauho, Pekka; Kalli, Heikki (Technical Univ. of Helsinki). Nucl. Sci. Eng., 33: 251-4(1968).

The discrete S_n method for solving the neutron transport problems is analyzed by discrete ordinates method; the directional neutron flux is approximated by a finite number of neutron rays with certain weights. The scalar flux is treated by S_n solution; discrete directions and weights are treated as free parameters. Two variational formulations are used: the integral transport equation; and self-adjoint variational formulation for monoenergetic Boltzmann equation by Toivanens. (auth)

C

42558 APPROXIMATE SOLUTION TO SPACE DEPENDENT KINETICS PROBLEMS. Suda, Nobuhide (Osaka Univ.). J. Nucl. Sci. Technol. (Tokyo), 5: 377-8(July 1968).

An approximation technique for solving space dependent kinetics problems is given. A synthesis technique is used for developing a one-group model in which the delayed neutron effect is ignored. The transient flux distribution in a uniform infinite slab core is determined; these results are compared with the exact solution. (M.L.S.)

C

48799 VARIATIONAL MULTICHANNEL SPACE-TIME SYNTHESIS METHOD FOR NONSEPARABLE REACTOR TRANSIENTS. Stacey, Weston M. Jr. (Knolls Atomic Power Lab., Schenectady, N. Y.). Nucl. Sci. Eng., 34: 45-56(Oct. 1968).

A multichannel space-time synthesis model for the calculation of nonseparable reactor transients is developed from a variational functional which admits expansion functions that are discontinuous in space and time. In each of many spatial regions, the flux during each interval of time is expanded in known functions of position with unknown expansion coefficients. The accuracy of the method, and its superiority with respect to the conventional single-channel space-time synthesis method, are demonstrated by several numerical examples. 22 references. (auth)

A

48803 NEW DERIVATION OF DISCRETE ORDINATE APPROXIMATIONS. Kaplan, Stanley (Univ. of Southern California, Los Angeles). Nucl. Sci. Eng., 34: 76-82(Oct. 1968).

It is shown that the idea of space-angle synthesis may be used to provide a new framework for the derivation of discrete ordinate methods. It is also shown that within this framework one can cleanly and systematically incorporate various stratagems aimed at overcoming the ray effects that plague discrete ordinate methods in two- and three-dimensional problems. 13 references. (auth)

B

50863 ITERATIVE-VARIATIONAL METHODS FOR NUMERICAL SOLUTION TO ELLIPTICAL PROBLEMS IN REACTOR THEORY. Albertoni, Sergio; Lunelli, Massimiliano; Muggioni, Gabriella (Applicazioni e Ricerche Scientifiche, Milan). pp 217-22 of Fisica del Reattore. Rome, Consiglio Nazionale delle Ricerche, 1966. (In Italian).

From Conference on Physics of Reactors, Milan. See CONF-469.

Methods for iterative-variational techniques for solution of n -dimensional diffusion equations for heterogeneous systems are described. Use of this technique as a basis for a computer program is discussed. (M.L.S.)

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C

1193 ENERGY OPTIMAL XENON SHUTDOWN. Lewins, Jeffery; Woodcock, Gordon; Babb, Albert (Univ. of Washington, Seattle). 20p. (CONF-660606-52). ORAU. Gmelin, AED-CONF-66-203-102.

From American Nuclear Society Meeting, Denver.

Pontryagin's optimum theory is applied to the problem of determining a flux shutdown program that limits Xe poisoning in a reactor while using the minimum nuclear energy on flux integral. We determine the complete adjoint solution and synthesize typical optimal trajectories. The analysis is carried through for both the unrestricted problem and the problem where the Xe density is restricted within the control period to some maximum acceptable poisoning. Much of the analysis is similar to the problem where the control time rather than the control energy is to be optimized. Our method of solution is new in utilizing a more straightforward optimum theorem and giving the full solution on the boundary of restriction. It is briefly indicated how the same analysis can be used to give a more rigorous proof of optimality in the time optimal problem. (auth)

F

7621 (AD-638024) COMPLEMENTARY VARIATIONAL PRINCIPLES IN NEUTRON DIFFUSION THEORY. Arthurs, A. M. (Wisconsin Univ., Madison, Theoretical Chemistry Inst.). Aug. 24, 1966. Contract DA-11-022-ORD-2059. 13p. (WIS-TCI-184). CFSTI \$3.00 cy, \$0.65 mn.

Complementary variational principles associated with neutron diffusion in solids are presented. The resulting formulas are used to derive new expressions which provide upper and lower bounds for the absorption probability. (auth)

B

7639 (WAPD-TM-610) VARIATIONAL PRINCIPLE FOR THE NEUTRON DIFFUSION EQUATION USING DISCONTINUOUS TRIAL FUNCTIONS. Buslik, A. J. (Bettis Atomic Power Lab., Pittsburgh, Pa.). Oct. 1966. Contract AT(11-1)-Gen-14. 20p. Dep. mn. CFSTI \$1.00 cy, \$0.50 mn.

A variational principle for the neutron diffusion equation which permits the use of discontinuous trial fluxes is presented. The principle is based on the method of Lagrange multipliers. Applications of the principle to problems of axial synthesis are given, and the principle is compared to that of Selengut and Wachspress. (auth)

C

7652 SOLUTION OF THE SPACE-TIME NEUTRON-GROUP DIFFUSION EQUATIONS BY A TIME-DISCONTINUOUS SYNTHESIS METHOD. Yasinsky, J. B. (Westinghouse Electric Corp., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 9: 458-7 (Oct.-Nov. 1966).

B

7911 APPLICATION OF SYNTHESIS TECHNIQUES TO THE CALCULATIONS OF THREE-DIMENSIONAL REACTIVITY COEFFICIENT DISTRIBUTIONS. Bear, J. L.; Judge, F. D.; Venerus, E. R. (General Electric Co., Schenectady, N. Y.). Trans. Amer. Nucl. Soc., 9: 470-1 (Oct.-Nov. 1966).

B, G

7912 FLUX SYNTHESIS USING MODIFIED GREEN'S FUNCTION MODES. Hooper, R. J.; Becker, M. (General Electric Co., Schenectady, N. Y.). Trans. Amer. Nucl. Soc., 9: 471 (Oct.-Nov. 1966).

C

10132 (WAPD-TM-641) TEST OF THE TIME SYNTHESIS APPROACH FOR THE SOLUTION OF REACTOR KINETICS PROBLEMS. Bewick, J. A.; Kaplan, S. (Bettis Atomic Power Lab., Pittsburgh, Pa.). Oct. 1966. Contract AT(11-1)-Gen-14. 33p. Dep. mn. CFSTI \$3.00 cy, \$0.65 mn.

A series of numerical experiments are reported in which time dependent diffusion problems are solved approximately by various versions of the modal analysis and time synthesis methods. (auth)

B

12140 (GEAP-4922) BISYN: A TWO DIMENSIONAL SYNTHESIS PROGRAM. Greebler, P.; Kelley, M. D.; Davis, R. A.; Keck, C. A.; Duncan, W. A. (General Electric Co., San Jose, Calif. Advanced Products Operation). July 15, 1965. Contract AT(04-3)-189. 150p. Dep. mn. CFSTI \$3.00 cy, \$0.65 mn.

A two-dimensional diffusion program (BISYN) for core analysis of fast reactors is described in which the two-dimensional fluxes and adjoints are synthesized from one-dimensional solutions using a noniterative technique. (D.C.W.)

F

17281 GENERALIZATION OF THE VARIATIONAL METHOD OF KAHAN, RIDEAU, AND ROUSSOPOULOS. II. A VARIATIONAL PRINCIPLE FOR LINEAR OPERATORS AND ITS APPLICATION TO NEUTRON-TRANSPORT THEORY. Koatin, Morton D. (Princeton Univ., N. J.); Brooks, Harvey. J. Math. Phys. (N. Y.), 8: 53-6 (Jan. 1967).

Additional applications of a generalized form of the variational method of Kahan, Rideau, and Roussopoulos are presented. Equations used in neutron transport theory, such as the spherical harmonics operator form of the Boltzmann equation, are derived from the generalized variational functional; and an interpretation of these operator equations in terms of flux- and source-generating operators is suggested. A relation between this variational method and the variational method of Lippmann and Schwinger is established, and it is shown that the least-square variational functional of Becker for linear equations can be derived from a generalized variational functional. (auth)

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G, B

17453 (KAPL-M-6588) SOME MATHEMATICAL PROPERTIES OF THE MULTICHANNEL VARIATIONAL SYNTHESIS EQUATIONS AND TWO-DIMENSIONAL SYNTHESIS NUMERICAL STUDIES. Wachspress, E. L. (Knolls Atomic Power Lab., Schenectady, N. Y.). Dec. 12, 1966. Contract W-31-109-eng-52. 31p. Dep. mn. CFSTI \$3.00 cy, \$0.65 mn.

It is proved that the matrix approximating the group diffusion operator ($-\nabla \cdot D\nabla + A$) in the multichannel variational synthesis mockup is positive definite. Some numerical studies of a highly nonseparable reactor are described. (auth)

F

21383 LIMITATION OF THE ROUSSOPOULOS VARIATIONAL PRINCIPLE? Pomraning, G. C. (General Atomic Div., General Dynamics Corp., San Diego, Calif.). Nucl. Sci. Eng., 28: 150-2(Apr. 1967).

It is shown that, in the application of the Roussopoulos variational principle, for a certain class of trial functions the designation of the error term as a second-order term is questionable, leading to doubts about the usual interpretation of the variational procedure. (D.C.W.)

F

21395 COMPLEMENTARY VARIATIONAL PRINCIPLES IN NEUTRON DIFFUSION THEORY. Arthurs, A. M. (Univ. of Wisconsin, Madison). Proc. Roy. Soc. (London), Ser. A, 298: 97-101(Mar. 28, 1967).

Complementary variational principles associated with neutron diffusion in solids are presented. The resulting formulas are used to derive new expressions which provide upper and lower bounds for the absorption probability. (auth)

G, B

21731 VARIATIONAL METHOD FOR MULTIGROUP NEUTRON FIELD SYNTHESIS IN MULTIZONE REACTORS. Khromov, V. V.; Slesarev, I. S.; Kuzmin, A. M. pp 11-32 of *Inzhenerno-Fizicheskie Voprosy Yadornykh Reaktorov*. Yurovov, L. N. (ed.). Moscow, Atomizdat, 1966. (In Russian).

The neutron field in a nuclear reactor can be described with a degree of accuracy by a multi-group set of kinetic equations which are written in vector-matrix form with a P_1 -approximation to avoid tedious calculations. A functional J can be composed on the basis of the neutron diffusion equations and the boundary conditions. The functional J is varied with respect to vector functions which satisfy the diffusion equation. The deviation of the functional from the exact value is a second-order difference compared with the deviation in the average vector function from the function which is a solution of the problem. The multigroup set of equations is reduced to a set of few-group equations by the use of the variational principle and by the low sensitivity of the functional to error in the vector functions. It is assumed that the space and energy components of the neutron flux and of the neutron worths are partially separable. The solutions give approximate values of integral fluxes and neutron worths in various reactor zones in a particular energy sub-space. The effective multiplication factor of the reactor is determined. (TFT)

F, A

25404 CANONICAL AND INVOLUTORY TRANSFORMATIONS OF THE VARIATIONAL PROBLEMS OF TRANSPORT THEORY. Kaplan, S.; Davis, James A. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 28: 166-76(May 1967).

The notions of canonical and involutory transformations from the calculus of variations are applied to neutron transport problems. It is shown that the variational formulations of Vladimirov, Selengut, Pomraning and Clark, and Davis are related through transformations of this type. It is pointed out that the pair of functionals identified through the involutory transformation are reciprocal in the sense that the minimum of one is the maximum of the other. The implications of this fact for the development of approximation methods are discussed. (auth)

F, C

27663 (AD-634843) NEUTRON IMPORTANCE AND VARIATIONAL FORMULATION IN REACTOR PHYSICS. Nozawa, Reikichi (Uppsala Univ. (Sweden)). Feb. 25, 1966. Contract AF 61(052)-874. 48p. (ARL-66-0120). CFSTI \$3.00 cy, \$0.65 mn.

The neutron importance is studied as a physical concept for a time-dependent system considering the case where successive detections are made as well as the final detection. Successive detections make the importance equation inhomogeneous. Based on the physical interpretation of neutron importance, a new formulation of variational principles is proposed in a manner applicable to time-dependent systems. The proposed variational functionals are to be understood not as Lagrangian densities as in classical or quantum theory of fields but as total detection data in experiments. As an application, a method of Green's functions is developed to improve successively a trial flux. The modification factor for the source term in the diffusion equation is derived in a most elementary way. (auth)

A

29841 ANGLE-SPACE SYNTHESIS: AN APPROACH TO TRANSPORT APPROXIMATIONS. Kaplan, S.; Davis, J. A.; Natelson, M. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 28: 364-75(June 1967). (WARD-T-1891).

A method is proposed for solving the transport equation in which the angular dependence is treated by expansion in specially tailored trial functions. The working equations of the method are derived and simple numerical examples presented. (auth)

G

30091 (ANL-7210) REACTOR PHYSICS DIVISION ANNUAL REPORT, JULY 1, 1965 TO JUNE 30, 1966. (Argonne National Lab., Ill.). Dec. 1966. Contract W-31-109-eng-38. 446p. Dep. CFSTI.

Brief descriptions of work on the following subjects are presented: fission properties and cross section data including fast neutron scattering studies, elastic neutron scattering from elements of intermediate weight, elastic neutron scattering from Mg and Si, fast neutron scattering from nuclei in the mass region $A = 95-130$, the interaction of fast neutrons with the 182, 184, and 186 isotopes of W, a search for fluctuations in the fission cross

section of ^{235}U , neutron flux measurements in the 10–200 keV region, (d,n) stripping reactions, fast neutron total cross sections using a monoenergetic source and an automatic facility, fast neutron energy degradation through the $(n,\gamma n')$ process, unitary models of nuclear resonance reactions, the ^{252}Cf fission neutron spectrum from 0.003–15.0 MeV, direct and absolute measurements of average yield of neutrons in the thermal fission of ^{235}U and spontaneous fission of ^{252}Cf , spontaneous fission half-lives of ^{242}Cm and ^{244}Cm ; thermal reactor physics including High Conversion (Hi-C) critical experiment, Hi-C uniform lattice calculations, initial critical experiments of the EBWR Pu recycle program, measurement of capture-to-fission ratios of ^{239}Pu and ^{241}Pu in the Pu loading of the EBWR, control rod evaluation for thermal and intermediate reactors, small reactivity measurements in the Argonne Thermal Source Reactor (ASTR), neutron beam spectra extracted from the High Flux Irradiation Reactor, Argonne Advanced Research Reactor (AARR) critical experiments—preface, AARR critical experiments—control blade worths, AARR critical experiments—prompt neutron lifetime measurements by the Rossi-alpha technique, AARR critical experiments—Cd ratio measurements, AARR critical experiments—activation and power distribution measurements, AARR critical experiments—void and material reactivity worths and temperature coefficients, AARR critical experiments—beam tube experiments, AARR critical experiments—startup source requirements and instrument response, AARR calculations—preface, AARR calculations—analysis of the critical experiments, AARR calculations—general reactor physics design analysis, AARR calculations—reactor physics characteristics of the ITC, AARR calculations—factors in optimization of experimental fluxes, AARR calculations—shield design analysis, AARR calculations—analyses of hypothetical accidents; fast reactor physics including the neutron energy spectrum in a dilute UC-fueled fast critical assembly, neutron spectra in depleted U, calculations of Na-void coefficients in large fast neutron carbide cores in assemblies No. 2 and 3 of ZPR-6, calculations of the effect of thin slab heterogeneities on the non-leakage reactivity component of Na voiding, non-linearity in the spectral component of Na void effect as a function of Na content, effect of parameter uncertainties on Na void effect and critical size of fast reactors, Doppler-effect measurements on a dilute carbide fast assembly—ZPR-6 assembly No. 4Z, measured physics parameters in a zoned fast UC core—ZPR-IV assembly No. 42, analysis of the uncertainties in the interpretation of zone loaded experiments, measurement of the spatial distribution of the importance of fission neutrons in ZPR-6 assembly No. 4Z, standard deviation of ion chamber current measurements in ZPR-6 assembly 4Z, measured reactivity removal rates in ZPR-6 assembly No. 4Z, the Argonne National Laboratory of ZPR-3 assembly No. 48, critical assembly comparison calculations using new cross section data, comparative neutronic characteristics of metal, oxide and carbide EBR-II driver fuels, the effect of fuel and blanket changes on the EBR-II flux, FAIR-T Core I fuel irradiation program and reference design, twenty-six group cross section set for W-based rocket systems, physics measurements in fast W rocket reactor critical experiments, measurement of space-dependent material worths in several ZPR-9 assemblies, rocket critical assemblies analysis, physics measurements in an operating fast breeder power reactor, further neutronic studies of the 1000 MW(e) metal-fueled fast breeder reactor, reactor physics calculations for a 10,000 MW(th) fast Na-cooled breeder, fast breeder reactors for water desalting, criteria for the density of monitoring points in large reactors; fast reactor safety including capabilities of the present TREAT facility core as a fast loop meltdown facility, meltdown experiments using the Mark I integral Na loop, analyses of single pin loop meltdown experiments,

properties of irradiated UO_2 pins prior to TREAT facility transients, photographic fast reactor safety experiments on irradiated oxide pins at the TREAT facility, transient in-pile tests on UO_2 -W cermet rocket fuel samples, design of the Mark II Integral TREAT facility Na loop, calibration mockup for the large loop test section for the TREAT facility, transient response of stand-off pressure transducer assemblies on the TREAT facility integral Na loops, extensible multi-purpose vacuum glove box, experimental results and improvements in the fast neutron hodoscope, the exact three-dimensional solution for thermoelastic stresses and displacements in finite and infinite tubes, transient vaporization of Na in reactor coolant channels, convective heat or mass transfer with phase changes, theoretical prediction of thermodynamic and transport properties of metal vapors, equation of state of reactor materials at high pressures and temperatures, a modified equation of state for hydrodynamic calculations in the AX-1 numerical program, properties of refractory ceramics at extremely high temperatures (UC liquid expansion), modification of the high temperature W filament furnace, failure pressures of thick-walled doubly-reinforced concrete cells, maximum permissible body burdens of Pu isotopes and resulting release criteria, fast reactor meltdown accident analysis code, PREAX; experimental physics techniques and facilities including a low geometry α counting chamber, absolute determination of fission rates in ^{235}U and ^{238}U and capture rates in ^{238}U by radiochemical techniques, precision fission rate measurements by fission track counting, solid-state Compton spectrometer for measurement of reactor γ spectra, feedback stabilization of nuclear counting channels, signal splitting into fast and slow channels, design and construction of an improved Mn bath counting system, low flux measurement of ^{235}U epithermal capture-to-fission ratio, reactor response to an oscillating neutron source, neutron fluxes required for activating probe materials, a code to permit fission product decay corrections without the use of a reference foil, determination of the k-constant for the Dy substitution method, additional calculations of the activation of spheres in a nonisotropic neutron flux, use of a small digital computer in data analysis and control of critical facilities, a Ge(Li) detector system for the measurement of γ -rays following inelastic neutron scattering, a multi-angle fast neutron time-of-flight system, multiple angle detector apparatus for neutron elastic scattering and polarisation measurements, multiple scattering correction, automated computed control of a fast neutron laboratory; reactor computation methods and theory including the Argonne Reactor Computation (ARC) system, the ARC system glossary, the Multigroup Constants Code (MC²), modification of THERMOS to generate transfer cross sections, generation of multigroup cross sections using a coupled MC²-THERMOS code, variation of thermal cross sections with buckling in consistent P1 and B1 calculations, development of a code to study fuel management, AMC—A Monte Carlo code, development and analysis of Monte Carlo methods, quasistatic treatment of space dependent reactor transients, space dependent kinetic calculations using the WIGLE code, reactor systems analysis and hybrid computers, computation of the coupled error function by continued fractions, treatment of source discontinuities in the solution of the diffusion equation, revision of the bulk shielding code MAC for the CDC-3600 computer, codes for analysis of elastic scattering angular distributions, multilevel cross sections for a fissionable isotope, the effect of interference on the resonance integral mixtures of Th and U, the effect of randomness on group cross sections, the chemical binding effects on the resonance line shapes of ^{238}U in a UO_2 lattice, equivalence between homogeneous and heterogeneous resonance integrals in cylindrical geometry, effect of the fluctuations in collision den-

sity on fast reactor Doppler effect calculations, an approximate calculation of space dependent flux using a variational principle, neutron-wave analysis; miscellaneous including energy spectrum of fast cosmic-ray neutrons near sea level, a CO₂ system for direct conversion of nuclear energy to coherent laser light, theory of plasma oscillations-generation of thermionic RF energy and interactions with DC, circulating shield reactor for space power, and improvised shutter design for the JANUS reactor. A total of 609 references is listed throughout the report. (M.L.S.)

G, B

30191 SYNTHESIS OF THREE-DIMENSIONAL FLUX SHAPES USING DISCONTINUOUS SETS OF TRIAL FUNCTIONS. Yasinsky, J. B.; Kaplan, S. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 28: 426-37(June 1967).

The method of flux synthesis is extended in a systematic way to allow the possibility of using different sets of trial functions in different axial zones. The necessary equations are derived in some detail and numerical examples are presented. The results of these examples are very satisfactory and suggest, therefore, that the synthesis procedure can be made much more useful and powerful by extending it in this way. In a more general context they suggest that the basic notation of deriving discontinuous-type approximation methods from an appropriate variational principle is a valid and very effective idea. (auth)

C

31950 (WAPD-TM-635) THE MULTIMODE SYNTHESIS APPROXIMATION FOR SPACE-TIME REACTOR DYNAMICS: AN APPRAISAL OF FINITE DIFFERENCING METHODS. Clarke, W. G.; Margolis, S. G. (Bettis Atomic Power Lab., Pittsburgh, Pa.). May 1967. Contract AT(11-1)-Gen-14. 76p. Dep. CFSTI.

An appraisal of possible finite differencing methods for the multimode time synthesis approximation to space-time reactor kinetics with emphasis on accuracy, stability, computer economy, and programming simplicity is presented. A FORTRAN IV digital program for the CDC-6600, MUMKIN-2, was used to perform these numerical studies and is described. The program is used to solve the multimode kinetics equations in the absence of reactivity feedback for up to nine trial functions and six delayed neutron groups. The program description includes an example of how multimode kinetics parameters can be generated by arithmetical operations on the synthesis edits of an existing one dimensional neutron diffusion program for up to four energy groups and 100 mesh points. Special features of the program include an automated data link for computing the multimode kinetics parameters ρ , β and Λ and a technique to achieve time synthesis with discontinuous sets of trial functions. (auth)

F

35554 (ANL-7320, pp 439-46) APPLICATION OF SPACE-ENERGY SYNTHESIS TO THE INTERPRETATION OF FAST MULTIZONE CRITICAL EXPERIMENTS. Storrer, F.; Chaumont, J. M. (Commissariat a l'Energie Atomique, Cadarache (France), Centre d'Etudes Nucleaires).

Information gained from multizoned critical experiments is

rather difficult to interpret and to extrapolate with a multigroup theoretical model. A much simpler model involving only few parameters has been tried out which would still give a good representation of the physical processes. This simple model is based on a two-group formalism, in which the groups overlap, derived from the diffusion equation by a variational procedure. Comparison between conventional multigroup and two-overlapping-group formalisms gives excellent agreement. It is hoped that with the help of this method the interpretation of multizoned critical experiments might be greatly eased. It is also suggested that synthesis methods in general be used to solve some special problems in fast reactor physics. A list of 21 references is included. (auth)

F

35607 THE USE OF NOETHER'S THEOREM IN REACTOR PHYSICS. Tavel, M. A.; Clancy, B. E.; Pomraning, G. C. (Brookhaven National Lab., Upton, N. Y.). Contract AT(30-2)-Gen-16. Nucl. Sci. Eng., 29: 58-66(July 1967). (BNL-10697).

A use of the variational method which has been neglected in reactor theory is discussed. This is the invariance theorem of E. Noether which has been widely utilized in other areas of mathematical physics. Following a derivation of the theorem, its use to obtain solutions of the time-independent diffusion equation is demonstrated. The theorem is used to construct a complete analogy between the time-dependent diffusion process and classical mechanics. Conservation laws arise in the construction of this analogy and their possible application is discussed. An analogy between the neutron diffusion equation and the time-dependent Schrodinger equation is also given. Several suggestions for generalizations of Noether's theorem for use in reactor theory are made. (auth)

A

38180 ANGULAR SYNTHESIS APPROXIMATION APPLIED TO THIN CRITICAL SLABS. Zwiibel, H. S. (Battelle-Pacific Northwest Lab., Richland, Wash.). Trans. Amer. Nucl. Soc., 10: 213-14(June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

A

38203 STRATEGY FOR THE APPLICATION OF SPACE-ANGLE SYNTHESIS. Natelson, M. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 10: 171-2(June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

B

38205 INITIAL STUDIES OF COLLAPSED GROUP-SPACE SYNTHESIS: ANOMALIES ARISING FROM THE USE OF ADJOINT WEIGHTING. Yasinsky, J. B.; Kaplan, S.; Jason, R. H. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 10: 172-3(June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

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F

38208 CANONICAL AND INVOLUTORY TRANSFORMATIONS OF VARIATIONAL PROBLEMS INVOLVING HIGHER DERIVATIVES. Kaplan, S. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 10: 175-6 (June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

C

38209 VARIATIONAL PRINCIPLE FOR THE DETERMINATION OF PSEUDOFUNDAMENTAL DECAY EIGENVALUES. Jenkins, J. D.; Daitch, P. B. (Rensselaer Polytechnic Inst., Troy, N. Y.). Trans. Amer. Nucl. Soc., 10: 177 (June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

B

38210 PERTURBATION METHOD THAT UTILIZES THE VARIATIONAL PRINCIPLES ASSOCIATED WITH THE NEUTRON-DIFFUSION EQUATION. Turley, Richard E. (Iowa State Univ., Ames). Trans. Amer. Nucl. Soc., 10: 177-8 (June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

C

38535 COMPUTATIONAL METHODS FOR SOLUTION OF REALISTIC SPACE-TIME DYNAMICS PROBLEMS. Kaplan, S. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 10: 203 (June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

C

38558 VARIATIONAL CALCULATION OF COMPLEX MODES OF XENON SPATIAL OSCILLATION. Buslik, A. J.; Weinreich, W. A. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 10: 257-8 (June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

B

38682 CALCULATION OF THE SODIUM-VOID EFFECT BY FLUX SYNTHESIS. Travelli, A.; Helm, F. (Argonne National Lab., Ill.). Trans. Amer. Nucl. Soc., 10: 275-6 (June 1967).

From 13th Annual Meeting of the American Nuclear Society, San Diego, Calif., June 11-15, 1967. See CONF-670602.

A

40364 THE EXTRAPOLATED ENDPOINT FOR THE MILNE PROBLEM. Pomraning, G. C. (General Atomic Div., General Dynamics Corp., San Diego, Calif.); Lathrop, K. D. Nucl. Sci. Eng., 29: 305-8 (Aug. 1967).

Upper and lower bounds on the extrapolated endpoint for a homogeneous source-free half-space with absorption and isotropic scattering are calculated by relating the extrapolated endpoint to a linear functional and applying variational methods. (D.C.W.)

F

40365 A DERIVATION OF VARIATIONAL PRINCIPLES FOR INHOMOGENEOUS EQUATIONS. Pomraning, G. C. (Brookhaven National Lab., Upton, N. Y.). Nucl. Sci. Eng., 29: 220-36 (Aug. 1967). (BNL-10514)

It is shown that variational principles need not be postulated and then correctness proved; they can, in fact, be derived, making their use more a matter of routine than ingenuity. A Lagrange multiplier technique is used to derive a second-order variational principle for estimating an arbitrary functional of the solution to an inhomogeneous equation. The relation of this principle to a functional Taylor series expansion and to elementary perturbation theory is established. A normalization independent second-order variational principle for an arbitrary functional is derived which reduces to the Schwinger principle if the functional is linear. Two higher-order variational principles are derived and are shown to be generalizations of the principles of Kostin and Brooks. The Lagrange multiplier technique is applied to the inhomogeneous Sturm-Liouville equation, which leads to a second-order variational principle for estimating an arbitrary functional which allows trial functions that are not continuous and do not satisfy the boundary conditions. This functional is of the type suggested by Buslik plus boundary terms. The differences between a variational principle which can only be used to estimate a functional of interest and one which also acts as a Lagrangian are discussed. (auth)

C

40609 THE APPLICATION OF THE CALCULUS OF VARIATIONS AND THE METHOD OF GREEN'S FUNCTION TO THE SOLUTION OF COUPLED CORE KINETICS EQUATIONS. Carter, Neal; Danofsky, Richard (Iowa State Univ., Ames). pp 249-69 of Coupled Reactor Kinetics. Chezem, C. G.; Koehler, W. H. (eds.). College Station, Tex., Texas A and M Press, 1967.

From American Nuclear Society, Coupled Reactor Kinetics Conference, College Station, Tex., Jan. 23-24, 1967. See CONF-670107.

Solution to the coupled core kinetics equations is presented in the form of a space-time modal analysis in terms of neutron flux. The flux is expanded in a series of space dependent functions having time dependent expansion coefficients. Sample calculations, utilizing the method, are given. 4 references. (M.L.S.)

C.

42756 SOLUTION OF THE SPACE-TIME NEUTRON GROUP DIFFUSION EQUATIONS BY A TIME-DISCONTINUOUS SYNTHESIS METHOD. Yasinsky, J. B. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 29: 381-91 (Sept. 1967).

A variational principle, which has as its stationary conditions

the direct and adjoint time-dependent group diffusion equations, is modified to admit time-discontinuous approximating functions. This extended principle is used to develop a synthesis approximation for the time-dependent group diffusion equations which permits the use of different sets of trial functions at different times during a transient analysis. The necessary equations are derived in detail, and two numerical examples are presented. These examples show that the time-discontinuous synthesis method is capable of constructing accurate space-time neutron fluxes, which vary smoothly in time, from spatial trial functions which are discontinuous in time. In addition, these examples display the potential of the new time synthesis for yielding computationally less expensive solutions than are possible with the time-continuous synthesis procedure. (auth)

F

42864 (WAPD-TM-659) CANONICAL AND INVOLUTORY TRANSFORMATIONS OF VARIATIONAL PROBLEMS INVOLVING SECOND DERIVATIVES: APPLICATION TO BEAMS AND PLATES. Kaplan, S. (Bettis Atomic Power Lab., Pittsburgh, Pa.). May 1967. Contract AT(11-1)-Gen-14. 54p. Dep. CFSTI.

The canonical and involutory transformations of the calculus of variations are applied to variational problems in which the integrand of the functional involves the second derivative of the argument function. One and two dimensional examples, i.e., the variational problems governing the deflection of beams and plates, are worked out in detail. (auth)

C

46656 APPROXIMATE SOLUTION TO THE TIME-DEPENDENT MULTIGROUP NEUTRON-DIFFUSION EQUATIONS USING A RESTRICTED VARIATIONAL PRINCIPLE. Robinson, James Conda. Knoxville, Tenn., Univ. of Tennessee, 1966. 115p.

Thesis.

For some reactors now in operation or being considered, it has become apparent that a time-dependent mathematical neutronic model which permits a shift in the neutron energy and/or spatial spectrum is required for the dynamic study of these systems. Models which describe the behavior of the neutrons in space, energy, and time are well-known, e.g., the multigroup neutron diffusion equations, but the solution to the resultant set of coupled nonlinear differential-integral equations must be approximate. The classical approximation is to assume that the neutron energy and spatial spectrums are independent of time. The purpose of this study was to develop an approximate solution for the time-dependent multigroup neutron diffusion equations with temperature-dependent parameters, thereby retaining a mechanism to permit a shift in the spatial or energy neutron spectrum during a transient. The nonlinear equations which describe the neutronic state of the system are linearized in a discrete fashion, i.e., the nuclear parameters are assumed constant over some spatial region and increment in time. As the calculation progresses in time, the nuclear parameters are adjusted as dictated from the neutronic and thermodynamic state of the system. An approximate solution is obtained for the set of linearized neutron diffusion equations by assuming the neu-

tron flux can be represented by linear combination of known spatial modes with time varying coefficients. The equations which determine the time varying coefficients are generated by applying a restricted variational principle. The spatial modes chosen to represent the flux are the Greens Function Modes. The delayed neutron precursors are explicitly taken into account by assigning the precursor equations as equations of constraint to the functional whose Euler equations include the system equations. Upon the application of the variational principle, the adjoint operator and adjoint function flux are introduced into the analysis. The solution to the adjoint equations is as equally difficult as the solution of the diffusion equations; therefore, a modal expansion is assumed for the adjoint flux. With the assumed solution for the forward and adjoint flux, the restricted variational principle leads to a coupled set of linear differential equations which determine the time-varying coefficients appearing in the assumed solutions. The number of such equations, in general, is the product of the number of modes by the number of groups appearing in the multigroup formalism. The number of equations can be reduced by assuming that different energy groups can be described by the same time varying coefficients. To demonstrate the validity of the approximate solution, several transients, which were performed on the TREAT reactor at Argonne National Laboratories, are simulated. The analytical results are presented graphically in the form of power traces which are compared with the experimental power traces. The analytical model is composed of five energy groups, up to three spatial modes, and six delayed neutron groups. The required data to carry out a simulation are the neutron multigroup parameters, delayed neutron fractions, precursor decay constants, and nuclide specifications. From the results, it is concluded that the model is adequate for short-term transients in thermal systems. Recommendations are set forth for useful extensions of the model to other problems of interest in reactor dynamics. (Dissert. Abstr.)

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F

1136 AN ANALOGY BETWEEN THE VARIATIONAL PRINCIPLES OF REACTOR THEORY AND THOSE OF CLASSICAL MECHANICS. Kaplan, S. (Westinghouse Electric Corp., West Mifflin, Pa.). Nucl. Sci. Eng., 23: 234-7 (Nov. 1965). (WAPD-T-1818).

A formal parallelism is shown to exist between two classical variational principles governing the time behavior of mechanical systems and two principles relating to the λ -mode eigenvalue problem of neutron group diffusion theory. By identifying the space variable with the time variable and space derivatives (gradients and divergences) with time derivatives, the "usual" variational principle of diffusion theory is shown to be analogous to Hamilton's principle and the diffusion equations are analogous to the Lagrange equations. Hamilton's canonical equations are then analogous to the diffusion equations in first-order form, and the analog of the principle involving the canonical integral is a principle closely related to one proposed recently by Selengut and Wachspress. (auth)

B

1140 BILINEAR AVERAGING FOR DIFFUSION THEORY PARAMETERS [Thesis]. Pitterle, Thomas Arthur. Madison, Wis., Univ. of Wisconsin, 1965. 126p.

Bilinear averaging of group parameters for use in diffusion-theory calculations is evaluated. This procedure averages energy-dependent cross sections over both neutron flux and adjoint spectra. Both perturbation theory and a variational principle are employed to obtain multigroup diffusion theory with bilinear averaged parameters as an approximation to the continuous energy-dependent P-1 equations. These derivations show that with bilinear averaging eigenvalues accurate to first-order errors in the fluxes used to average the cross sections can be obtained. It is also shown that bilinear averaging represents a consistent approximation to the energy-dependent equations for perturbation theory calculations. Extensive numerical calculations were made to compare flux and bilinear averaging procedures for diffusion theory calculations. A few perturbation theory calculations were also carried out. (TCO)

A

2978 VARIATIONAL VACUUM BOUNDARY CONDITIONS FOR A P_N APPROXIMATION. Davis, James A. (Westinghouse Electric Co., Pittsburgh). Trans. Amer. Nucl. Soc., 8: 484-5 (Nov. 1965).

F

2979 HIGHER ORDER VARIATIONAL PRINCIPLES. Selengut, D. S. (General Electric Co., Schenectady, N. Y.). Trans. Amer. Nucl. Soc., 8: 485 (Nov. 1965).

A

2980 A NUMERICAL COMPARISON OF HIGHER ORDER VARIATIONAL PRINCIPLES. Schreiner, Sheldon (Univ. of California, Berkeley); Selengut, D. S. Trans. Amer. Nucl. Soc., 8: 485-6 (Nov. 1965).

A

2990 TRANSPORT SYNTHESIS. Davis, James A.; Kaplan, Stanley (Westinghouse Electric Corp., Pittsburgh). Trans. Amer. Nucl. Soc., 8: 509 (Nov. 1965).

F

3331 AN ANALOGY BETWEEN THE VARIATIONAL PRINCIPLES OF REACTOR THEORY AND THOSE OF CLASSICAL MECHANICS. Kaplan, S. (Westinghouse Electric Corp., Pittsburgh). Trans. Amer. Nucl. Soc., 8: 535-6 (Nov. 1965).

D

3454 FEW LONG-TIME-INCREMENT DEPLETION CALCULATIONS FOR APPROXIMATING BY SYNTHESIS A MANY SHORT-TIME-INCREMENT DEPLETION CALCULATION. Flanagan, Charles A. (Westinghouse Electric Corp., Pittsburgh). Trans. Amer. Nucl. Soc., 8: 518-19 (Nov. 1965).

A

6264 (WAPD-T-1849) TRANSPORT SYNTHESIS. Davis, James A.; Kaplan, Stanley (Bettis Atomic Power Lab., Pittsburgh, Pa.). Sept. 1965. Contract AT(11-1)-Gen-14. 15p. (CONF-651101-45). Dep. mn; CFSTI \$1.00 cy, \$0.50 mn.

From 13th Conference on Remote Systems Technology, Washington, D. C.

A FORTRAN program (TRANSY) is developed to obtain synthesized solutions to the S_2 or S_4 equations for a monoenergetic x-y problem. The TRANSY synthesis is based on the discrete-ordinate method. The purpose of the synthesis is to investigate the feasibility of synthesizing solutions to high-order approximations to the transport equation. (T.F.H.)

B, G

8038 (KAPL-3095) VARIATIONAL MULTICHANNEL SYNTHESIS WITH DISCONTINUOUS TRIAL FUNCTIONS. Wachspress, E. L.; Becker, M. (Knolls Atomic Power Lab., Schenectady, N. Y.). Nov. 1965. Contract W-31-109-eng-52. 55p. Dep. mn; CFSTI \$3.00 cy, \$0.50 mn.

A variational formulation of multichannel synthesis

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is developed for neutron diffusion computations through use of a functional which admits discontinuous trial functions. Detailed equations are given for a particular choice of trial functions. A variational depletion formalism is described for conducting reactor life studies with a multi-channel synthesis calculation model. (auth)

F

8353 CALCULATION OF FLUX DISTRIBUTION AND EIGENVALUE OF ARBITRARILY SHAPED REACTORS BY MEANS OF THE CALCULUS OF VARIATIONS. Unger, H. Atomkernenergie, 10: 413-14(Nov.-Dec. 1965). (In German).

An investigation was made on the effect of the reactor shape on the essential characteristics of the reactor, e.g., specific power and surface heat flux density. The problem consisted of calculating and comparing the neutron flux distribution and criticality of bare, homogeneous simple-bordered reactors of arbitrary shape in order to obtain the reactor form associated with the optimal reactor. A method is presented for the solution of the problem by means of a variational calculation. (I.B.S.)

C, B

9957 THE APPLICATION OF VARIATIONAL METHODS TO NEUTRON THERMALIZATION PROBLEMS [Thesis]. Razminas, Robert B. Gainesville, Fla., Univ. of Florida, 1965. 174p.

Methods based on the variational method, VM, and the generalized variational method, GVM, are applied to the calculation of the thermalization parameters \bar{D}_0 and C_0 associated with the asymptotic time eigenvalue through the power series $\lambda - \alpha_0 + \bar{D}_0 B^2 - C_0 B^4$ for the pulsed neutron experiment in crystalline media. A procedure is given for determining only the lowest or asymptotic eigenvalue without recourse to parametric methods using selected values of the geometric buckling, B^2 . The trial functions selected to represent the energy spectrum are the Laguerre polynomials of order 1 weighted by the energy Maxwellian. However, the general development of the analysis is not restricted to these functions. Any suitable complete set of polynomials could be utilized. Several examples incorporating low ordered trial functions are given to provide a detailed description of the mechanics of adapting the VM and the GVM to the problem and also to allow an analytic comparison of the parameters obtained from the two methods. General equations from which thermalization parameters can be obtained using high ordered trial functions are developed for both the VM and the GVM. Also included are tabulations of pertinent Laguerre weighted integrals (up to 18 polynomials) of energy-dependent scattering cross-section parameters obtained from the scattering kernel $\Sigma_s(E' \rightarrow E)$ selected to represent the scattering interactions that may occur in the particular medium. The calculation of the various integrals and hence the thermalization parameters is transformed to a discrete energy representa-

tion. The scattering kernels used in the calculations for the crystalline media beryllium and graphite are based on the energy exchange model formulated by Parks, and tables of the kernel data for the selected energy points used in this study are included. The influence of the energy mesh size upon the values of the thermalization parameters is investigated and discussed. Also discussed is the convergence behavior of the parameters obtained using both the VM and the GVM. The values of the thermalization parameters \bar{D}_0 and C_0 obtained in this study for beryllium and graphite respectively are: 1.33×10^5 cm²/sec and 6.47×10^5 cm⁴/sec; 2.15×10^5 cm²/sec and 16.06×10^5 cm⁴/sec. (Dissertation Abstr.)

G

12368 (WAPD-T-1737) AN EVALUATION OF A THREE-DIMENSIONAL FLUX SYNTHESIS METHOD AS A NUCLEAR DESIGN TOOL. Flanagan, C. A.; Smith, F. E.; Bogar, G. F.; Rutherford, C. H. (Bettis Atomic Power Lab., West Mifflin, Pa.). Oct. 1964. Contract AT(11-1)-Gen-14. 20p. (CONF-654-100). Dep. mn. CFSTI \$1.00 cy, \$0.50 mn.

From American Nuclear Society Meeting, San Francisco, Nov.-Dec. 1964.

Research is described towards evaluating the use of the weighted residual flux synthesis method in three-dimensional nuclear design calculations. The evaluation includes comparison of synthesis calculations with direct three-dimensional calculations, a comparison of results from synthesis calculations with experiment, and finally an examination of the practicality of applying the method to depletion calculations involving considerable geometric complexity. (auth)

12369 EVALUATION OF VARIATIONAL METHOD ACCURACY. Erykalov, A. N. At. Energ. (USSR), 19: 462-3(Nov. 1965). (In Russian).

A study is made of a variational method for determining critical reactor dimensions and neutron distributions, with which the upper and lower limits of the eigenvalue of the hermitian operator can be calculated. The high accuracy of the eigenvalues obtained using simple approximations is demonstrated for two examples. The accuracy is computed for a single-group diffusion equation for the case of constant cross section and cosinusoidal flux distribution and the case of constant diffusion constant and cosinusoidal absorption and fission cross-section distribution. (M.J.T.)

B

15666 (CONF-660204-4) VARIATIONAL METHODS IN THE CALCULATION OF SPATIALLY DEPENDENT SPECTRA. Francis, N. C. (Knolls Atomic Power Lab., Schenectady, N. Y.). [1965]. Contract W-31-109-eng-52. 30p. Dep. mn. CFSTI \$2.00 cy, \$0.50 mn.

From American Nuclear Society Meeting, San Diego, Calif.

A discussion is presented of variational methods for cases in which the quantities studied are stationary with respect to the variable of interest. Resonance capture is not studied. After an introduction of the spatially independent problem, a specific thermal-energy-range spatially dependent neutron problem is discussed. The method is then applied to calculations of group constants for a multi-group Monte Carlo method, multigroup space-energy problems in slab geometry, and the use of overlapping groups in diffusion theory. (T.F.H.)

B

16033 A VARIATIONAL APPROACH TO THE EFFECT OF BOUNDARY PERTURBATIONS WHEN THE MULTI-GROUP DIFFUSION EQUATIONS APPLY. Spinks, N. (Australian Atomic Energy Commission, Lucas Heights, New South Wales). 14p. (CONF-650602-71). ORINS. Gmelin, AED-CONF-65-125-107.

From American Nuclear Society 11th Annual Meeting, Gatlinburg, Tenn.

Variational principles are given which allow perturbations of the surface defining the boundary of a region in which the multi-group diffusion equations apply. The perturbation may take the form of a change in boundary conditions or a change in the boundary shape. The principles are useful for determining the reactivity worth of control rods when the shape of the rod makes a direct calculation difficult. In addition, from a single flux and adjoint flux calculation, the worths of a host of control rods of slightly different compositions (boundary conditions) and slightly different shapes and sizes can be determined. The application of the principle to the output from finite difference computer codes is discussed. (auth)

G

17656 NUMERICAL SOLUTION OF THE DIFFUSION EQUATIONS WITH THE KANTOROVICH METHOD. Galligani, I. (CCR, Euratom, Ispra, Italy); Giorcelli, M. *Energia Nucl. (Milan)*, 13: 73-81 (Feb. 1966).

The time-independent two dimensional neutron diffusion equations are solved with a variational method (Kantorovich's method). This method is particularly convenient when the neutron flux is a smooth function along one of the two directions. A comparison between this method and the finite difference method is made by means of some numerical examples. (auth)

F

18059 A VARIATIONAL PRINCIPLE FOR RATIOS IN CRITICAL SYSTEMS. Lewins, J. (Univ. of Washington, Seattle). *J. Nucl. Energy, Pt. A and B*, 20: 141-3 (Feb. 1966).

A variational principle from which arbitrary ratios in

critical systems can be calculated in a stationary manner is presented. This principle enables the perturbation expression to be derived concisely using a compact operator notation in a generalized X variable (position and velocity) that incorporates the usual exact and approximate models of reactor theory. (B.G.D.)

A

18068 PARTICULAR SOLUTIONS OF THE ENERGY-DEPENDENT BOLTZMANN EQUATION. Mueller, Karl-Heinz (CCR EURATOM, Ispra, Italy). *Nukleonik*, 8: 38-40 (Jan. 1966).

The Carleman-Kupradse method for solving a coupled set of integral equations with sectional holomorphic kernels combined with a method based on a variation procedure, of removing the energy from the field of free variables, allows the construction of particular solutions of the neutron transport equation. The nuclear data enter the calculation only after they are weighted and integrated over the energy variables (E,E'). Thus, the procedure allows the use of cross sections in a numerical form (e.g., the usual energy group representation). An approximate solution can be found from a linear system of algebraic equations. (auth)

F

21957 A NUMERICAL STUDY OF THE METHOD OF WEIGHTED RESIDUALS. Pomraning, G. C. *Nucl. Sci. Eng.*, 24: 291-301 (Mar. 1966).

The variational method and region-balance method, both special cases of the more general method of weighted residuals, are each used as the formalism to develop a spatial expansion of the diffusion equation for two problems. These are (1) a spatially dependent spectrum problem for the purpose of computing the self-shielding in the ²³⁹Pu resonance and (2) a simple one-dimensional eigenvalue problem. In both instances numerical results indicate that the variational method is more accurate than the region-balance method. Of particular interest is the variational spatial-expansion approach to the eigenvalue problem. This may be a useful method for deriving a set of difference equations for the multigroup diffusion equation in that it should lead to an accurate representation of the flux with a relatively small number of mesh points. (auth)

B

22387 VARIATIONAL ITERATIVE METHODS FOR ELLIPTICAL PROBLEMS IN THE THEORY OF NUCLEAR REACTORS. Albertoni, S.; Lunelli, M.; Maggioni, G. (Univ., Milan). *Atti Seminar. Mat. Fis. Univ. Modena*, 14: 169-201 (1965). (in Italian).

A variational iterative method is derived for the solution of elliptical problems in nuclear reactor theory. The variational bases for the re-solution of transmission problems are given. The iterative-variational technique used for the numerical solution is examined. Numerical examples are then given. (J.S.R.)

G

22399 VARIATION PRINCIPLES FOR THE CRITICAL FLUX CURVES OF REFLECTED REACTORS. Brettenhuber, L. (Technische Hochschule, Graz). *Z. Angew. Phys.*, 20: 267-71(1966). (In German).

New minimum principles and stationary expressions are given and discussed for the boundary conditions $\phi = 0$ and $\partial\phi/\partial n + (1/d)\phi = 0$ for the equation $\Delta\phi + B^2\phi = 0$. It is suggested to fulfill conditions at boundaries and interfaces only in the form of natural boundary conditions. Thus, the use of discontinuous trial functions is encouraged, yielding greater flexibility and better numerical approximations in the mean. Boundary conditions are, nevertheless, satisfied in the limit. (auth)

F

24302 (GA-6273) A NUMERICAL STUDY OF THE METHOD OF WEIGHTED RESIDUALS. Pomraning, G. C. (General Atomic, San Diego, Calif. John Jay Hopkins Lab. for Pure and Applied Science). Mar. 29, 1965. Contract AT(04-3)-187. 40p. Dep. mn. CFSTI \$2.00 cy, \$0.50 mn.

The variational method and the region-balance method, both special cases of the more general method of weighted residuals, are each used as the formalism to develop a spatial expansion of the diffusion equation for two problems. These are a spatially dependent spectrum problem for the purpose of computing the self-shielding in the ^{240}Pu resonance and a simple one-dimensional eigenvalue problem. In both instances, the numerical results indicate that the variational method is more accurate than the region-balance method. Of particular interest is the variational spatial expansion approach to the eigenvalue problem. This may be a useful method for deriving a set of difference equations for the multigroup diffusion equation in that it should lead to an accurate representation of the flux with a relatively small number of mesh points. (auth)

F

24714 IMPROVED CALCULATION METHODS FOR RATIOS APPEARING IN REACTOR STATICS. Lewins, Jeffery (Univ. of Washington, Seattle). *Nucl. Energy*, 69-76(Mar. 1966).

Variational and perturbation expressions are developed from which to calculate ratios of interest in either critical systems or in sub-critical systems having a source. The several adjoint functions employed are related to the physical concept of the importance. The emphasis is placed on the calculation of the adjoint functions by iterative methods that are already standard in reactor codes. The expressions given can improve the accuracy of results while requiring substantially less computing time than conventional direct calculations. (auth)

C, A

28235 (NAA-SR-11821) SPACE-TIME FLUX SYNTHESIS METHODS FOR THE APPROXIMATE SOLUTION OF TIME-DEPENDENT BOLTZMANN NEUTRON TRANS-

PORT EQUATION. Luco, V. (Atomics International, Canoga Park, Calif.). May 25, 1966. Contract AT(11-1)-Gen-8. 37p. Dep. mn. CFSTI \$2.00 cy, \$0.50 mn.

Space-time flux synthesis methods for the approximate solution of the time-dependent Boltzmann neutron transport equation are formulated. The variational and the Galerkin technique are used. The space-dependent part of the solution is obtained with the DTF-11 program—an S_n -type solution of the transport equation. Temperature feedback effects are considered in the formulation. The formulation presented here is adapted to the description of rapid transients following a large reactivity input, but the method can be easily modified to cover other reactor problems where a space-time flux description is necessary. It should be useful whenever the situation requires the transport approximation for the description of the neutron flux. (auth)

A

30116 VARIATIONAL VACUUM BOUNDARY CONDITIONS FOR A P_N APPROXIMATION. Davis, James A. *Nucl. Sci. Eng.*, 25: 189-97(1966). (WAPD-T-1848)

Approximate vacuum boundary conditions for a P_N approximation are obtained by variational methods. Two stationary principles are proposed, one having what is called "odd" Marshak conditions as its natural boundary conditions, and the other having "even" Marshak conditions as its natural boundary conditions. The principles are valid for arbitrary geometry. The odd Marshak conditions are seen to be suitable for an odd-order P_N approximation and the even Marshak conditions for an even-order P_N approximation. The odd Marshak conditions are precisely the conditions obtained by Vladimirov from an extremum principle in which certain restrictions are imposed on the source and scattering. The present treatment contains no such restrictions. (auth)

A

30332 SELF-ADJOINT VARIATIONAL PRINCIPLE FOR DERIVING VACUUM AND INTERFACE BOUNDARY CONDITIONS IN THE SPHERICAL HARMONICS METHOD. Toivanen, Timo. *Nucl. Sci. Eng.*, 25: 275-84(July 1966).

By the technique of splitting the total directional flux into even and odd portions in angle, the stationary monoenergetic Boltzmann equation with arbitrary collision kernel and with arbitrary external directional source of a general geometry is symmetrized to a self-adjoint form. The continuity and boundary conditions for the resulting self-adjoint integro-differential equation are explicitly constructed. A variational principle is then set up by devising a self-adjoint Lagrangian whose minimum property is equivalent to the symmetrized Boltzmann equation with the associated continuity and boundary conditions. The developed variational principle contains no arbitrariness and is used for deriving unique variational boundary conditions for the P_l approximation of the spherical harmonics method. It is shown, for a general geometry, that applying the semidirect variational method with an angle-independent trial function yields, without any physical reasoning, the correct P_l differential equation and the corresponding no-return-current boundary condition. (auth)

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C

30852 VARIATIONAL METHOD FOR PROMPT NEUTRON KINETICS. Koehler, Walter H. (Kernforschungszentrum, Karlsruhe, Ger.). Nukleonik, 8: 203-15(May 1966).

It is shown that the Euler-Lagrange equations of a variational principle are the multigroup diffusion equations for prompt neutrons and their adjoint equations. The Kantorovich method is used to get approximate solutions for the nonseparable flux. The method is applied to a step change in configuration of the fast core of a coupled fast-thermal reactor. (auth)

B

34490 DETERMINATION OF EIGENVALUES OF THE DIFFUSION EQUATION BY VARIATIONAL TECHNIQUES. Bolstad, J. W.; Selengut, D. S. (Univ. of California, Berkeley). Trans. Amer. Nucl. Soc., 9: 192-3(June 1966).

A

34491 VARIATIONAL ESTIMATES OF DIFFUSION AND EXTRAPOLATION LENGTHS WITH P_1 SCATTERING. Henryson II, H.; Selengut, D. S. (Univ. of California, Berkeley). Trans. Amer. Nucl. Soc., 9: 193-4(June 1966).

A, F

34492 UNIFYING APPROACH TO VARIATIONAL FORMULATIONS OF THE TRANSPORT EQUATION. Kaplan, Stanley; Davis, James A. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 9: 194(June 1966).

B

34494 VARIATIONAL TECHNIQUES APPLIED TO THE INFINITE-MEDIUM SLOWING DOWN OF NEUTRONS. Hawk, Thomas L.; Fenech, Henri (Massachusetts Inst. of Tech., Cambridge). Trans. Amer. Nucl. Soc., 9: 195-6(June 1966).

A

34495 SPACE ANGLE SYNTHESIS: AN APPROACH TO TRANSPORT APPROXIMATIONS. Kaplan, Stanley; Davis, James A. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 9: 196-7(June 1966).

A

34496 NUMERICAL EXAMPLES OF SPACE ANGLE SYNTHESIS. Natelson, M.; Kaplan, S. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 9: 197-8(June 1966).

G, B

34953 SYNTHESIS OF THREE-DIMENSIONAL FLUX SHAPES USING DISCONTINUOUS SETS OF TRIAL FUNCTIONS. Yasinsky, J. B.; Kaplan, S. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 9: 198-9(June 1966).

B

34954 VARIATIONAL PRINCIPLE FOR THE NEUTRON-DIFFUSION EQUATION USING DISCONTINUOUS TRIAL FUNCTIONS. Buslik, A. J. (Bettis Atomic Power Lab., West Mifflin, Pa.). Trans. Amer. Nucl. Soc., 9: 199(June 1966).

G, B

36143 (WAPD-T-1885) A VARIATIONAL PRINCIPLE FOR THE NEUTRON DIFFUSION EQUATION USING DISCONTINUOUS TRIAL FUNCTIONS. Buslik, A. J. (Bettis Atomic Power Lab., Pittsburgh, Pa.). Dec. 1965. Contract AT-11-1-GEN-14. 14p. (CONF-660606-13). Dep. mn. CFSTI \$1.00 cy, \$0.50 mn.

From American Nuclear Society Meeting, Denver. Selengut and Wachspress have developed a variational principle for the neutron diffusion equation which permits the use of discontinuous trial functions. Another variational principle is described which is useful with a wider class of trial functions. (M.O.W.)

A

40360 (WAPD-T-1890) ANGLE-SPACE SYNTHESIS: AN APPROACH TO TRANSPORT APPROXIMATIONS. Kaplan, S.; Davis, J. A.; Natelson, M. (Bettis Atomic Power Lab., Pittsburgh, Pa.). May 1966. Contract AT(11-1)-Gen-14. 49p. (CONF-660606-14). Dep. mn. CFSTI \$2.00 cy, \$0.50 mn.

From American Nuclear Society Meeting, Denver. Three new ideas are combined to produce an analytical approach to solution of the neutron transport equation in realistic multidimensional reactor geometries. The method is of the expansion approach type in which the flux is represented by a linear combination of known functions of angle with mixing coefficients which are unknown functions of space. The mechanics of the new approach are developed in detail and some initial numerical examples are described. (H.D.R.)

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F

41409 VARIATIONAL DESCRIPTION OF DISSIPATIVE PROCESSES. Pomraning, G. C. (General Atomic Div., General Dynamics Corp., San Diego, Calif.). J. Nucl. Energy: Pt. A and B, 20: 617-34(Aug. 1966).

The Roussepoulos variational formalism is applied to equations with a first-order time derivative. The resulting functional is a very general variational characterization of dissipative processes in that it admits trial functions which do not satisfy the equation of interest, do not satisfy the boundary conditions, and are not continuous. It is shown that the previous functionals presented in the literature for this type of problem are special cases of this more general functional. The use of this variational principle to estimate a rather general class of characteristics of interest is discussed. It is also pointed out that this general functional is not unique and arguments are given to deal with this non-uniqueness. The variational description of the Sturm-Liouville equation is considered in this same generality. This leads to a generalization (to a more complete class of admissible trial functions) of the classical Rayleigh quotient for estimating eigenvalues. (auth)

G

46967 SYNTHESIS METHODS IN REACTOR ANALYSIS. Kaplan, S. (Westinghouse Electric Corp., West Mifflin, Pa.). Advances Nucl. Sci. Technol., 3: 233-66(1966). (WAPD-T-1810).

A collection of methods for solving various problems in reactor theory is described. The unifying idea is to reduce the number of independent variables by constructing an approximate solution in the form of a linear combination of known functions of one or more of the variables, with the coefficients of combination being functions only of the remaining variables. One of the important applications of the described approximation methods is solving the neutron group diffusion equations in three dimensions with great spatial detail. (H.D.R.)

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F, C

1848 TIME-DEPENDENT VARIATIONAL PRINCIPLES FOR NONCONSERVATIVE SYSTEMS. Jeffrey Lewins (Royal Engineers, Glasgow). Nucl. Sci. Eng., 20: 517-20 (Dec. 1964).

Two variational principles are discussed for time-dependent problems in reactor physics. The first is a stationary expression for the meter reading at a given time, the second a stationary expression for the integral of the meter reading up to a given time. Both the principles, unlike conventional Lagrangians extended to time-dependent nonconservative systems, have the advantage of requiring trial functions to be exact only at one end of the time interval of interest. Either may be generalized to account for nonlinearities. The second principle reduces to the first by making a suitable identification, while the first principle in turn reduces to a well-known and powerful variational principle for the steady state. (auth)

F

2713 THE PRINCIPLES AND APPLICATIONS OF VARIATIONAL METHODS. Martin Becker. Cambridge, Massachusetts, Massachusetts Institute of Technology, 1964. 130p. \$5.00.

Fundamental variational calculus techniques are described. As an example, these are applied to the calculation of fuel depletion in nuclear reactors. (T.F.H.)

A, H

3196 ANALYTICAL REPRESENTATION OF NEUTRON FLUX IN PLATE-LAYERS. Karl-Heinz Mueller (CCR-EURATOM, Ispra, Italy). Nukleonik, 6: 271-4 (Sept. 1964). (In German)

The direct method of variation calculation, applied to the Boltzmann equation for neutron transport, allowed the construction of analytical expressions, by means of which the neutron distribution in the reactor could be described in clearly arranged ways. For the one-dimensional plate layers, three useful ways for the solution of shielding questions are shown and performed. (tr-auth)

C

3547 VARIATIONAL ANALYSIS OF A NON-LINEAR KINETICS PROBLEM. David S. Selengut (Knolls Atomic Power Lab., Schenectady, N. Y.). Trans. Am. Nucl. Soc., 7: 256-7 (Nov. 1964).

D

3562 APPLICATION OF MULTICHANNEL SYNTHESIS TO TWO-DIMENSIONAL DEPLETION CALCULATIONS. W. H. Turner, F. C. Merriman, and E. C. Hansen (Knolls Atomic Power Lab., Schenectady, N. Y.). Trans. Am. Nucl. Soc., 7: 290 (Nov. 1964).

D

3563 LEAST SQUARES VARIATIONAL ANALYSIS OF THE BURNUP PROBLEM. Martin Becker (Knolls Atomic Power Lab., Schenectady, N. Y.) and Henri Fenech. Trans. Am. Nucl. Soc., 7: 290-1 (Nov. 1964).

G, D

5436 (WAPD-TM-468) TNT02—A THREE-DIMENSIONAL NEUTRON FLUX SYNTHESIS AND DEPLETION CALCULATION COMPUTER PROGRAM. C. A. Flanagan, R. J. Breen, O. J. Marlowe, and A. J. Vigilotti (Westinghouse Electric Corp. Bettis Atomic Power Lab., West Mifflin, Penna.). Aug. 1964. Contract AT-11-1-GEN-14. 103p. Dep.(mn); \$4.00(cy), 2(mn) OTS.

TNT02 is a three-dimensional neutron flux synthesis and depletion code for Philco 2000 made up by TNT2A, which determines the integrals of the products of two-dimensional trial and weight functions, TNT2B, which performs the depletion calculations, and TNT2C, which performs re-depletion calculations for detailed power distributions. (R.E.U.)

F

8383 GENERALIZATION OF THE VARIATIONAL METHOD OF KAHAN, RIDEAU, AND ROUSSOPOULOS AND ITS APPLICATION TO NEUTRON TRANSPORT THEORY. Morton D. Kostin (Princeton Univ., N. J.) and Harvey Brooks. J. Math. Phys. (N. Y.), 5: 1691-1700 (Dec. 1964).

The variational method of Kahan, Rideau, and Roussopoulos (KRR) frequently used in neutron transport theory to estimate weighted averages is extended and generalized. In the KRR variational method a first variation in the trial functions produces a second variation in the estimate of the weighted average. Two generalized variational functionals which depend on trial operators instead of trial functions are given. A first variation in the trial operators produces an nth variation in the estimate of the weighted average when an nth order generalized variational functional is used. Both perturbation theory and the KRR variational method are derived as special cases of the generalized variational method. Several examples including calculations of transport equation spatial moments using diffusion equation solutions as trial operators are studied with good results. (auth)

G

12455 (EUR-2174.1) SOLUZIONE NUMERICA DI ALCUNI PROBLEMI BIDIMENSIONALI DELLA DIFFUSIONE NEUTRONICA COL METODO VARIAZIONALE DI RITZ. (Numerical Solution of Some Two-Dimensional Diffusion Neutronical Problems by the Variational Ritz Method). I. Galligani (European Atomic Energy Community. Joint Nuclear Research Center, Ispra, Italy and

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European Atomic Energy Community. European Scientific Data Processing Center, Ispra, Italy). 1964. 82p. Dep. (mn).

The approximate solutions of the homogeneous problem and of the inhomogeneous problem of the multigroup neutron diffusion theory may be obtained by searching for the functions which render suitable stationary functionals. These functions are calculated by the Ritz method. Some numerical experiments are given in order to compare the Ritz method with the finite difference method on two-dimensional diffusion problems. (auth)

D

14803 (CONF-651-26) LEAST SQUARES VARIATIONAL ANALYSIS OF THE BURNUP PROBLEM. Martin Becker (Knolls Atomic Power Lab., Schenectady, N. Y.) and Henri Fenech (Massachusetts Inst. of Tech., Cambridge). [1964]. Contract W-31-109-eng-52. 14p. Dep. (mn); \$1.00(cy), 1(mn) OTS.

From American Nuclear Society Meeting, San Francisco, Nov.-Dec. 1964.

A homogeneous, bare-slab reactor is considered that has a single fuel and is controlled by a variable uniform poison. One-group theory is used to describe the neutron behavior. The fuel burnup in this reactor is calculated by least-squares variational techniques. The FEVER computer code is used for the calculations. (T.F.H.)

B

17234 A MODIFIED KANTOROWITSCH METHOD FOR THE DIFFUSION THEORETICAL CALCULATION OF REACTORS WITH COMPLICATED GEOMETRIES. PART I. Hermann Schaeffler (Technische Hochschule, Stuttgart). Nukleonik, 6: 384-95(Dec. 1964). (In German)

For reactors with complicated geometry, as is present, for example, on account of any control elements (rods or plates) introduced in the reactor, the diffusion equation is not soluble analytically. An approximation method must be used. The modified Kantorowitsch method makes it possible to calculate a large class of such non-separable geometries. It differs from the approximation methods used in the literature in that it makes it possible to obtain very accurate results at relatively low expenditure. The calculation time amounts, in the modified method, to only a fraction of the time necessary for other methods. It is possible to calculate complicated problems in practicable calculation time. The method depends on a new formulation by means of which the variation problem equivalent to

the differential equation problem can be solved approximately. The method was demonstrated on the example of a bare cylinder reactor with central partially inserted black control rod and a reactor with re-entering angles. (tr-auth)

B

17235 A MODIFIED KANTOROWITSCH METHOD FOR THE DIFFUSION THEORETICAL CALCULATION OF REACTORS WITH COMPLICATED GEOMETRIES. PART II. Hermann Schaeffler (Technische Hochschule, Stuttgart). Nukleonik, 6: 395-405(Dec. 1964). (In German)

The modified method was discussed for some special control rod positions. It was applied to a reactor with re-entering angles and expanded to some further geometries. Numerical results were given for the eigenvalues and compared with values from the literature. (tr-auth)

G, B

21638 (CONF-650501-4) VARIATIONAL SYNTHESIS WITH DISCONTINUOUS TRIAL FUNCTIONS. E. L. Wachspress and M. Becker (Knolls Atomic Power Lab., Schenectady, N. Y.). Feb. 1965. Contract W-31-109-eng-52. 22p. Dep.(mn); \$1.00(cy), 1(mn) CFSTI.

From the Application of Computing Methods to Reactor Problems, Argonne, Ill.

The multichannel flux synthesis method for treating neutron-diffusion problems in several dimensions is considered. In this method, regions are represented by nodes, and nodal difference equations are derived with the aid of the lower-dimensional results. By suitably choosing a functional and trial functions, nodal difference equations are derived from variational principles. Discontinuous trial functions are used. Methods for obviating the difficulties presented by these discontinuities are discussed. (T.F.H.)

G

21654 SOLUTION OF A TWO-DIMENSIONAL NEUTRON DIFFUSION PROBLEM BY FLUX SYNTHESIS. Peter Killian (AEG-Kernenergieanlagen, Frankfurt am Main). Nukleonik, 6: 340-4(Nov. 1964). (In German)

The calculation of the two-dimensional distribution of the thermal neutron flux in the large cell of a water-moderated reactor was reduced with the help of a variation method to the calculation of the one-dimensional flux distribution. The position-dependent superposition of a second one-dimensional asymptotic flux gives a good approximation of the flux distribution. Iterations improved the results only slightly. (tr-auth)

C

27397 (WANL-TNR-133) VARI-QUIR—A TWO-DIMENSIONAL TIME-DEPENDENT MULTI-GROUP DIFFUSION CODE. J. W. Riese and G. Collier (Westinghouse Electric Corp., Pittsburgh, Pa. Astronuclear Lab.). Sept. 1963. 93p.

A variational approximation is developed to solve the time-dependent neutron diffusion equations. The method is incorporated into a computer code, which allows up to 4 energy groups and 6 precursors. Some sample results are presented. (auth)

B

27407 A MORE STRAIGHTFORWARD USE OF VARIATIONAL PRINCIPLES WITH BOUNDARY CONDITIONS. Harvey Amster (Univ. of California, Berkeley). Nucl. Sci. Eng., 22: 254-9(1965).

A previous treatment (Nucl. Sci. Eng., 16: 147-54(1963)) of one-group diffusion theory for a homogeneous slab is reviewed for the special situation in which scattering is isotropic and no neutrons enter the medium outside its surfaces. (C.E.S.)

A

27408 THE TREATMENT OF BOUNDARY TERMS IN A VARIATIONAL PRINCIPLE CHARACTERIZING TRANSPORT THEORY. G. C. Pomraning (General Atomic Div., General Dynamics Corp., San Diego, Calif.). Nucl. Sci. Eng., 22: 259-61(1965).

A variational treatment (H. Amster, Nucl. Sci., Eng., 22: 255 (1965)) of the diffusion theory boundary conditions is considered. This treatment is purported to be more correct than a variational treatment by Pomraning and Clark (Nucl. Sci. Eng., 16: 147 (1963)). It is pointed out that the treatment of the diffusion theory (or higher order P_N) boundary conditions is, by necessity, arbitrary (even within the framework of the variational method) and that the two treatments are different in their aims. (C.E.S.)

A

27409 ANSWER TO POMRANING'S REBUTTAL ON VARIATIONAL BOUNDARY CONDITIONS. Harvey Amster (Univ. of California, Berkeley). Nucl. Sci. Eng., 22: 262 (1965).

Rebuttal to the arguments given in "The Treatment of Boundary Terms in a Variational Principle Characterizing Transport Theory" (Nucl. Sci. Eng., 22: 259 (1965)) is presented. (C.E.S.)

C

29206 THE APPLICATION OF TWO VARIATIONAL TECHNIQUES TO THE ANALYSIS OF THE PULSED-NEUTRON EXPERIMENT. Robert B. Rasminas, G. R. Dalton, and M. J. Ohanian (Univ. of Florida, Gainesville). Trans. Am. Nucl. Soc., 8: 274(May 1965).

C

29529 COMPARISON OF THREE VARIATIONAL METHODS APPLIED TO A KINETICS PROBLEM. H. Fenech and V. Orphan (Massachusetts Inst. of Tech., Cambridge). Trans. Am. Nucl. Soc., 8: 223-4(May 1965).

A

31430 STUDY ON AN ANALYTICAL REPRESENTATION OF THE NEUTRON FLUX IN PLATE LAMINATIONS. Karl-Heinz Mueller (CCR Euratom, Ispra, Italy). Nukleonik, 6: 271-4(1964). (In German) (EUR-876.d)

The direct variation-calculus method, if applied to the Boltzmann neutron transport equation, enables analytical expressions to be formulated by means of which the neutron distribution in the reactor can be clearly described. For the unidimensional plate lamination, three methods which lend themselves readily to the solution of shielding questions are adduced and applied. (auth)

A

33196 (NUS-R-435) THE ASYMPTOTIC ANGULAR DEPENDENT LEAKAGE SPECTRUM OF THERMAL NEUTRONS. Kladnik, R. (Nuklearni institut "Jozef Stefan," Ljubljana (Yugoslavia)). Aug. 1964. 19p. Dep.(mn).

The application of the variational method in velocity-dependent transport theory is described for the case of a pulsed infinite slab. The extrapolated endpoint and the buckling as functions of the slab thickness are calculated from a given transcendental equation. The expression for the angular and energy distribution of leakage neutrons is improved by one iteration of the integral transport equation. The results are discussed for three different scattering models: monatomic gas, Nelkin water, and the Egelstaff model. It is found that Nelkin's model gives a better estimate for the integral constants, whereas the Egelstaff's law is more suitable for the differential spectrum calculations. (auth)

C

33207 ON THE INCLUSION OF BOUNDARY TERMS IN TIME-DEPENDENT SYNTHESIS TECHNIQUES. Becker, Martin (General Electric Co., Schenectady, N. Y.). Nucl. Sci. Eng., 22: 385-6(July 1965).

In connection with the application of synthesis techniques to various time-dependent problems, a variational principle was previously presented for linear time-dependent group-diffusion theory. However, the principle is not stationary with respect to arbitrary variations in the functions involved. It is shown that this difficulty can be avoided by inclusion of appropriate boundary terms in the functional. (D.C.W.)

C

47945 (ANL-7050, pp 159-77) VARIATIONAL METHODS FOR THE SOLUTION OF PROBLEMS OF REACTOR KINETICS. Galligani, Ilio (European Atomic

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Energy Community, Ispra (Italy). Joint Nuclear Research Center).

The solution of the time-dependent multigroup, multiregion diffusion equations may be obtained by searching for the functions that render stationary a particular functional. These functions are determined numerically by Kantorovich's method and by Euler's method. With the Kantorovich method the flux and the adjoint flux are expanded in a set of space-dependent trial functions with time-dependent coefficients. These coefficients are determined by solving with a "one-step method" a system of ordinary differential equations. The Euler method consists in searching for the functions that render stationary the above functional in the class of piecewise linear functions. These functions satisfy a large system of ordinary differential equations and are determined numerically by three different methods, the "implicit" method, the "implicitly explicit" method, and the Saul'yev "alternating" method. A comparison between the Kantorovich method and the Euler method is made by means of some numerical examples. (auth)

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A

994 A NEW ASYMPTOTIC DIFFUSION THEORY. G. C. Pomraning and M. Clark, Jr. (Massachusetts Inst. of Tech., Cambridge). Nucl. Sci. Eng., 17: 227-33(Oct. 1963).

The variational formalism is used to derive from the monoenergetic Boltzmann equation a diffusion theory with the asymptotic transport diffusion coefficient. By considering an interface between two media as the limiting case of a medium with continuously varying properties, the boundary conditions are found to be continuity of current and a specified discontinuity in the scalar flux. The variational formalism gives the linear extrapolation distance for a pure scatterer accurate to within one-half percent. Numerical comparisons with classical (P-1) diffusion theory for a cell calculation indicate that the variational diffusion theory is significantly more accurate; the accuracy appears to be comparable with that of the P-3 method. (auth)

F

1180 ON THE DERIVATION OF A VARIATIONAL PRINCIPLE FOR LINEAR SYSTEMS. D. S. Selengut (Knolls Atomic Power Lab., Schenectady, N. Y.). Nucl. Sci. Eng., 17: 310-11(Oct. 1963). (TID-18628)

A specified physical system, such as a reactor, whose state is described by a function of the relevant phase space coordinates is considered. A derivation is outlined that enables one to take a given linear theory and immediately obtain a lagrangian whose stationary property is equivalent to the equations of the theory and that constitutes a variational principle for the estimation of an arbitrary linear functional of the state of the system. (C.E.S.)

B

3121 A VARIATIONAL PRINCIPLE FOR HETEROGENEOUS RESONANCE CAPTURE. Paul F. Gast (General Electric Co., Hanford, Wash.). Trans. Am. Nucl. Soc., 6: 271-2(Nov. 1963).

D, B

3220 APPLICATION OF SYNTHESIS APPROXIMATIONS TO THREE-DIMENSIONAL DEPLETION CALCULATIONS AND TO CELL THEORY. S. Kaplan and O. J. Marlowe (Westinghouse Electric Corp., Pittsburgh). Trans. Am. Nucl. Soc., 6: 254-5(Nov. 1963).

C

7659 (CONF-187-27) APPLICATION OF THE VARIATIONAL METHOD TO THE CALCULATION OF THE TIME DEPENDENCE OF THE NEUTRON FLUX IN SMALL PULSED SLABS, CYLINDERS AND SPHERES. F. D. Judge and P. B. Daltch (Rensselaer Polytechnic Inst., Troy, N. Y.). [nd]. Contract W-31-109-eng-52. 20p.

From American Nuclear Society Meeting, New York, Nov. 1963.

The variational method is applied to the monoenergetic time dependent transport equation to obtain a simple relation for the asymptotic decay constant in small pulsed assemblies. The results indicate that flat trial functions may be a reasonable representation of the flux distributions in the thin slab limit. This approach is superior to many of the usual transport approximations. (auth)

B

15131 (EUR-532.e) A MULTI-DIMENSIONAL, MULTI-GROUP, MULTIREGION, NEUTRON DIFFUSION CODE (ITERATIVE-VARIATIONAL APPROACH). M. Lunelli and G. Maggioni (Milan. Università). 1964. 170p.

The program LOUISE III, in FORTRAN for the IBM 7090, calculates the criticality constant and critical fluxes of multiregion (up to 25) reactors in the multigroup (up to 10) approximation. Calculations can be performed in up to 7 dimensions, and an iterative-variational technique is used. The diffusion differential equations considered are included, together with a sample problem and the FORTRAN listing. LOUISE III has given results in agreement with WHIRLAWAY, while requiring a much shorter execution time. (auth)

B, G

15148 (WAPD-TM-377) EQUATIONS AND PROGRAMS FOR SOLUTIONS OF THE NEUTRON GROUP DIFFUSION EQUATIONS BY SYNTHESIS APPROXIMATIONS. S. Kaplan, O. J. Marlowe, and W. R. Cadwell (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh). Dec. 1963. Contract AT(11-1)-Gen-14. 71p.

A method for constructing approximate solutions to the three-dimensional group diffusion equations using only one- and two-dimensional programs is presented. The computer programs that have been developed to carry out this construction are described, and instructions are given for their use. (auth)

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C

15154 APPLICATION OF SYNTHESIS TECHNIQUES TO PROBLEMS INVOLVING TIME DEPENDENCE. S. Kaplan, O. J. Marlowe, and J. Bewick (Westinghouse Electric Corp., Pittsburgh). Nucl. Sci. Eng., 18: 163-76(Feb. 1964). (WAPD-T-1599)

A method described previously for synthesizing three-dimensional flux distributions is extended to reactor kinetics problems and to lifetime studies. The method is outlined, and some numerical examples are presented. The results of these show the method to be a practical way of solving time-dependent reactor problems with a detailed spatial model. (auth)

B

15468 (EUR-217.d(Pt. I and II)) UBER LOSUNGSVERFAHREN FUR DIE BOLTZMANN-GLEICHUNG. TEIL I AND II. (The Application of the Direct Method of the Variational-Calculus. Part I and II). K.-H. Mueller (European Atomic Energy Community. Joint Nuclear Research Center, Ispra, Italy). 1964. 30p.

A functional is given in which the P_1 approximation to the Boltzmann equation of neutron dynamics is in the form of Euler equations. The requirement for the related extreme values is thus equivalent to the differential equation system of the P_1 approximation. The direct method of calculating variations can be used to solve the variation problem. An exclusive system of selection functions, suitable for concrete reactor problems, provides a clear and simple method for calculating criticality and neutron flux. The result of each of these three procedures is called reduced Boltzmann-eq. This matrix is an integro-diff.eq. in (1) velocity and time, (2) space and time, and (3) time. A successive application of transformation (2) after (1) or (1) after (2) is possible. Examples illustrate the essential steps of the consideration. (auth)

A

18858 NEUTRON TRANSPORT IN CYLINDRICAL RODS. K. Bingham Cady and Melville Clark, Jr. (Massachusetts Inst. of Tech., Cambridge). Nucl. Sci. Eng. 18: 491-507(Apr. 1964).

A calculational method for Boltzmann's one-velocity, isotropic scattering transport equation is developed for cylindrical rods. The starting point is Peierl's integral equation, and the technique may be interpreted as a moments method or a variational method. Numerical results in the form of graphs are given for a set of standard problems. These problems include volume sources, surface

A

18859 AN IMPROVED FREE-SURFACE BOUNDARY CONDITION FOR THE P-3 APPROXIMATION. G. C. Pomraning (General Electric Co., Pleasanton, Calif.). Nucl. Sci. Eng., 18: 528-30(Apr. 1964).

Variational calculus is used to obtain an approximate free-surface boundary condition for the P_3 approximation in the monoenergetic neutron transport equations. Slab geometry is considered for simplicity. (C.E.S.)

sources, and the critical rod problem. For arbitrary, axially symmetric sources inside or outside the rod, a knowledge of the uncollided flux is sufficient to determine the escape probability from the rod in terms of these standard problems. (auth)

A

24573 VARIATIONAL BOUNDARY CONDITIONS FOR THE SPHERICAL HARMONICS APPROXIMATION TO THE NEUTRON TRANSPORT EQUATION. G. C. Pomraning (General Electric Co., Pleasanton, Calif.). Ann. Phys. (N. Y.), 27: 193-215(Apr. 1964).

It is shown that the monoenergetic neutron transport equation and the associated boundary conditions can be characterized by a Lagrangian. A proper choice of the trial function for this Lagrangian leads to the widely used spherical harmonics approximation as the Euler-Lagrange equations. A set of boundary conditions for the spherical harmonic equations is the result of the logical application of the variational method. These variational boundary conditions appear to be significantly more accurate than the boundary conditions presently in general use. For example, the use of the variational boundary conditions at a free surface reduces the error (compared with the boundary conditions presently used) in the linear extrapolation distance for the Milne problem by several factors. In particular, the P-1 (diffusion) approximation yields a value of 0.7071 (in units of mean free paths) and the P-3 approximation yields a value of 0.7118, both comparing quite favorably with the exact value of 0.7104. (auth)

A

28532 SPATIAL EXPANSION OF THE TRANSPORT EQUATION. G. C. Pomraning and M. Clark, Jr. (Massachusetts Inst. of Tech., Cambridge). J. Nucl. Energy, Pt. A & B, 18: 191-209(Apr. 1964).

The solution of the monoenergetic transport equation in slab geometry is expanded in terms of known spatial functions and unknown angular coefficients. Two general for-

malisms—a variational method and a moment-conservation method—are developed to obtain the necessary equations for the angular coefficients. The latter formalism is emphasized since it always yields neutron conservation. Further, it is shown that this formalism treats exactly any incident flux boundary condition, including the discontinuous vacuum boundary condition. For small systems with a highly peaked directional flux, a spatial expansion of the directional flux in integer powers of z (the spatial co-ordinate) is shown to yield extremely good results. Thus, a polynomial expansion forms a complement to the widely used angular expansions, such as the P-N (spherical harmonic) method, which are most accurate for large systems with an almost isotropic directional flux. To emphasize the fact that the formalisms developed are applicable to any spatial expansion functions, the penetration (of a normal beam) problem is considered with exponential expansion functions. The analysis is shown to reduce to the exact transport result in all known limits. (auth)

B

28536 A VARIATIONAL PROCEDURE FOR CALCULATING HIGH-ENERGY, FEW-GROUP, SPATIALLY DEPENDENT SPECTRA. P. A. Ombrellaro (Knolls Atomic Power Lab., Schenectady, N. Y.). *Trans. Am. Nucl. Soc.*, 7: 10-11 (June 1964).

F

28537 LEAST-SQUARES VARIATIONAL METHODS. Martin Becker and Henri Fenech (Massachusetts Inst. of Tech., Cambridge). *Trans. Am. Nucl. Soc.*, 7: 11-12 (June 1964).

C

28539 A VARIATIONAL TECHNIQUE FOR SPACE-TIME NEUTRON DIFFUSION. J. W. Riese (Westinghouse Electric Corp., Pittsburgh). *Trans. Am. Nucl. Soc.*, 7: 22-3 (June 1964).

A

28966 APPLICATION OF THE DIRECT METHOD OF VARIATIONAL CALCULUS TO THE BOLTZMANN EQUATION. K. H. Mueller (Euratom, Ispra, Italy). *Trans. Am. Nucl. Soc.*, 7: 9-10 (June 1964).

G

28968 APPLICATION OF FLUX SYNTHESIS TO PARAMETRIC-OPTIMIZATION STUDIES. N. J. Curlee, Jr. (Westinghouse Electric Corp., Pittsburgh). *Trans. Am. Nucl. Soc.*, 7: 12 (June 1964).

A

30701 ON THE VARIATIONAL METHOD APPLIED TO THE MONOENERGETIC BOLTZMANN EQUATION. Janusz R. Mika (Inst. of Nuclear Research, Warsaw). *Nucl. Sci. Eng.*, 19: 377-8 (July 1964).

The separation of the non-self-adjointness from the monoenergetic Boltzmann equation is discussed. The proper boundary terms that yield the boundary conditions corresponding to the adjoint Boltzmann equation are derived. (C.E.S.)

B, E

32714 (CONF-446-7) A VARIATIONAL PROCEDURE FOR CALCULATING HIGH-ENERGY FEW-GROUP SPATIALLY DEPENDENT SPECTRA. P. A. Ombrellaro (Knolls Atomic Power Lab., Schenectady, N. Y.). [June 1964]. Contract W-31-109-eng-52. 14p.

From American Nuclear Society 10th Annual Meeting, Philadelphia, June 1964.

A fast method that provides the accuracy comparable to multigroup methods was developed for reactor calculations in the 10 Mev to 0.625 ev range. The energy range is divided into three discrete lethargy groups in which a different set of two overlapping spectra is used for each group. Results are compared with IBM 704 multigroup PIMG calculations. (R.E.U.)

C, A

32721 A VARIATIONAL APPROACH TO THE TIME-DEPENDENT SLAB. R. Kladrnik (Kernforschungszentrum, Karlsruhe, Ger.). *Nukleonik*, 6: 147-53 (May 1964).

The asymptotic behavior of a neutron pulse injected into an infinite slab of finite thickness is discussed. For illustration, a monokinetic transport of neutrons with isotropic scattering was chosen. Two integrals of the transport equation and the variational method are used in order to obtain the dependence of the extrapolated end point upon the slab thickness. The extrapolated end point x_0 is obtained as the largest positive root of a given transcendental equation. It is suggested that the asymptotic mode may exist within the slab no matter how thin the slab is. The expression for the angular distribution of the leaking neutrons was improved by one iteration of the integral transport equation. The results show a characteristic angular peaking of the neutrons leaking from slabs of extremely small thicknesses. (auth)

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35921 (HW-SA-3190) CONTROL OPTIMIZATION BY VARIATIONAL CALCULUS. Albert L. Ruiz (General Electric Co. Hanford Atomic Products Operation, Richland, Wash.). Sept. 10, 1963. Contract AT(45-1)-1350. 24p.

The use of dynamic programming to synthesize an optimal control system when the performance index is expressed as an integral leads to results that have been shown to be equivalent to the use of the calculus of variations with natural boundary conditions. With this approach, the extension to other types of boundary conditions is not obvious. On the other hand, the use of the calculus of variations leads to results for all types of boundary conditions, as long as the variables are continuous. For a linear process and quadratic performance index, the derived control law is the same for all cases of admissible boundary conditions, and the feedback factors and gains are calculated from sets of first order differential equations that are of the same general form in each case, although differing in detail. For a nonlinear process or a nonquadratic performance index, the method gives a symbolic expression for the optimal control law. The evaluation of this expression for particular problems depends on one's ability to solve, exactly or approximately, a nonlinear differential equation. (auth)

A
40466 (APED-4342) SPATIAL EXPANSION OF THE TRANSPORT EQUATION. G. C. Pomraning and M. Clark, Jr. (General Electric Co. Vallecitos Atomic Lab., Pleasanton, Calif.). Mar. 25, 1964. 39p. (63-APE-16)

The solution of the monoenergetic transport equation in slab geometry is expanded in terms of known spatial functions and unknown angular coefficients. Two general formalisms, a variational method and a moment conservation method, are developed to obtain the necessary equations for the angular coefficients. The latter formalism is emphasized since it always yields neutron conservation. Further, it is shown that this formalism treats exactly any incident flux boundary condition, including the discontinuous vacuum boundary condition. For small systems with a highly peaked directional flux, a spatial expansion of the directional flux in integer powers of z (the spatial coordinate) is shown to yield extremely good results. Thus a polynomial expansion forms a complement to the widely used angular expansions, such as the P-N (spherical harmonic) method, which are most accurate for large systems with an almost isotropic directional flux. To emphasize the fact that the formalisms developed are applicable to any spatial expansion functions, the penetration (of a normal beam) problem is considered with exponential expansion functions. The analysis is shown to reduce to the exact transport result in all known limits. (auth)

B, E

42597 (TID-7050(p.229-76)) FAST GROUP FITTED CONSTANTS IN FEW-GROUP THEORY. P. A. Ombrellaro (Knolls Atomic Power Lab., Schenectady, N. Y.).

Two schemes for calculating fast few-group constants for core depletion calculations are described. The fitted cross-section scheme expresses the fast group constants in terms of effective microscopic cross sections, which are obtained by fitting the simple group-constant formulas to the group constants obtained from a multigroup program such as MUFT. The second scheme uses a variational method to solve the energy-dependent diffusion equations based on the assumption that, for a given composition, the flux spectrum is adequately given as a linear combination of two flux base spectra, and similarly for the current spectrum, where the bases are chosen to represent soft and hard spectra. The energy range from 10 Mev to 0.625 ev is represented in terms of lethargy, and few-group constants are obtained for a three-, two-, and one-group representation. (R.E.U.)

B, E

42605 (TID-7050(p.494-519)) A TWO-MODE VARIATIONAL PROCEDURE FOR CALCULATING THERMAL-DIFFUSION THEORY PARAMETERS. P. A. Ombrellaro (Knolls Atomic Power Lab., Schenectady, N. Y.).

The approximation of variations in the thermal group constants with depletion by a two-mode variational procedure is discussed. The procedure was programmed in FORTRAN for IBM 704 (SPG), and the results were compared with group constants obtained from the SOFOCATE program. (R.E.U.)

G, B

42927 (TID-7050(p.657-678)) SINGLE-CHANNEL SYNTHESIS. J. H. Leonard (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh).

The development of three-dimensional flux and power distributions in a reactor by combining one- and two-dimensional calculations for different portions of the reactor core is discussed. Synthesis (combination) involving a single axial function is known as single-channel synthesis, and is discussed. (R.E.U.)

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G, B

42928 (TID-7050(p.678-710)) MULTICHANNEL SYNTHESIS. E. L. Wachspress (Knolls Atomic Power Lab., Schenectady, N. Y.).

The development of three-dimensional flux and power distributions in a reactor by combining one- and two-dimensional calculations for different portions of the reactor core is discussed. Synthesis involving simultaneous computation of several flux distributions parallel to the z axis is discussed. (R.E.U.)

G, B

42929 (TID-7050(p.710-28)) VARIATIONAL SYNTHESIS. S. Kaplan (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh).

The development of three-dimensional flux and power distributions in a reactor by combining one- and two-dimensional calculations for different portions of the reactor core is discussed. It is assumed that the radial flux distributions are not independent of the axial flux distributions and that they vary continuously with the axial position. A set of trial radial distributions, representative of the possible extreme radial distributions, are combined by means of a variational principle applied to the flux distributions over the entire core to give the optimum combination of these trial radial flux distributions as a function of axial position, thus yielding a continuous distribution of flux in three dimensions. (R.E.U.)

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F

3745 WINNER OF THE MARK MILLS AWARD—A GENERALIZED VARIATIONAL METHOD FOR REACTOR ANALYSIS. Alfred L. Mowery, Jr. and Raymond L. Murray (North Carolina State Coll., Raleigh). *Trans. Am. Nucl. Soc.*, 5: 347(Nov. 1962).

C

3820 SPACE AND TIME SYNTHESIS BY THE VARIATIONAL METHOD. S. Kaplan (Westinghouse Electric Corp., Pittsburgh). *Trans. Am. Nucl. Soc.*, 5: 412-13(Nov. 1962).

F

3821 THE CONSTRUCTION OF APPROXIMATE THEORIES BY VARIATIONAL METHODS. D. S. Selengut (Knolls Atomic Power Lab., Schenectady, N. Y.). *Trans. Am. Nucl. Soc.*, 5: 413-14(Nov. 1962).

E

4043 VARIATIONAL METHODS IN NEUTRON THERMALIZATION. D. S. Selengut (Knolls Atomic Power Lab., Schenectady, N. Y.). p.162-86 of "Neutron Physics." New York, Academic Press, 1962.

The general formalism and the estimation of adjoint trial functions of the variational methods in neutron thermalization are treated. Some typical problems in neutron thermalization are discussed: the determination of the neutron spectrum in an infinite medium with a uniform source; the spectrum of a decaying pulse of neutrons as a function of time in a finite block of material and the associated thermalization time constant; and the rethermalization cross section and asymptotic decay length at large distances from a localized static source of thermal neutrons. The variational method provides a unified means of calculating these quantities in terms of a few parameters with direct physical interpretations. (W.D.M.)

E, B

5298 (KAPL-2220) A VARIATIONAL PROCEDURE FOR CALCULATING FAST FEW-GROUP CONSTANTS. P. A. Ombrellaro and F. D. Federighi (Knolls Atomic Power Lab., Schenectady, N. Y.). Aug. 10, 1962. Contract

W-31-109-eng-52. 46p.

A variational procedure for calculating fast-energy few-group constants is described. For a given medium, the method permits the flux and current solutions of the Boltzmann equation, treated according to P-1 slowing-down theory in each group of a few-group scheme as a linear combination of two base flux spectra to be expressed, the group flux to be obtained, and as a linear combination of two base current spectra, the group current to be obtained. The coefficients for combining the base spectra are provided by the theory and depend only on the concentrations of the component elements of the medium. Once the flux and current spectra in each group are calculated, the group constants for the medium can be easily calculated from base-flux-spectra weighted library microscopic cross sections. Group constants calculated in this manner agree well with those obtained from the MUFT 5 program. (auth)

C

9863 (WAPD-T-1534) SPACE AND TIME SYNTHESIS BY THE VARIATIONAL METHOD. S. Kaplan (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh). Oct. 1962. Contract AT(11-1)-Gen-14. 29p.

Approximation methods being developed for the solution of few-group diffusion equations for core designs requiring solutions to large-size problems are described. These methods use the process of synthesis by constructing approximate solutions to multi-dimensional problems out of solutions to fewer dimensional problems. Both space and time synthesis by the variational method are described. (M.C.G.)

F

11972 A GENERALIZED VARIATIONAL METHOD FOR REACTOR ANALYSIS. Alfred L. Mowery, Jr. and Raymond L. Murray (North Carolina State Coll., Raleigh). *Nucl. Sci. Eng.*, 14: 401-13(Dec. 1962).

A technique designated as the generalized variational method (GVM) is described. The analysis is based on the variational approach and is an outgrowth of investigations in the hypercircle method. In essence, the GVM consists of considering the trial functions that appear symmetrically (quadratically) in a positive-semidefinite variational principle as independent functions. A proposition is proved to demonstrate generally that the approximate eigenvalue

obtained from the GVM is at least as accurate as the geometric average of the associated approximate eigenvalues. Also, a conjecture is proposed that the accuracy of the generalized variational eigenvalue is comparable to that of a variational result employing a trial function incorporating the dimensionality of both associated trial functions. The application of the GVM to the perturbation-variational method yields results that establish the method. The generalized method completes the perturbation-variational method by providing the even-order approximate results. For illustration, the GVM is employed to solve a bare reactor with a grey control sheet. Using Rayleigh-Ritz optimized cosine series and optimized pyramid functions as associated solutions, the generalized variational eigenvalue accuracy indicates the effective combination of the dimensionalities of the associated trial functions. (auth)

C

21317 (KAPL-2217) THE SPACE-TIME NEUTRON KINETICS BY A VARIATIONAL METHOD WITH APPLICATION TO POWER REACTOR DYNAMICS. D. E. Dougherty (Knolls Atomic Power Lab., Schenectady, N. Y.). Dec. 26, 1962. Contract W-31-109-eng-52. 57p.

By use of the semidirect variational method, the time and space dependent neutron kinetic equations were reduced to a system of integro-differential equations in the time domain. The state vector of these equations defines the time-dependent coefficients for a set of space modes given by a modal expansion of the neutron flux. The accuracy of this method of analysis is arbitrary, and solutions are available for reactor configurations as complex as the present state of the art allows steady-state determinations. To avoid the difficult task of determining orthogonal eigenfunctions for a modal expansion in a complex geometry, nonorthogonal Green's Function modes are developed from an approximate solution of the integral form of the multi-group kinetic equations. Since an exact steady-state solution of the one-group diffusion model is available for a bare two-region slab reactor, this solution was compared with the results obtained by both orthogonal and nonorthogonal modal expansions. The basis of comparison was the asymptotic region buckling for a critical (steady-state) two-region reactor, following a perturbation of the reactor material properties in either of the core regions. A two-term expansion by Green's Function modes was clearly superior to an expansion in which the first two orthogonal modes for asymmetric perturbations of the core properties were used. Combining power coefficient dynamics with the reduced space-dependent neutron kinetic equations yields

a state equation that is in general nonlinear. By linearizing these equations and using the transfer function formalism, the transformed equation of state for the modal coefficients for a generalized power coefficient is obtained. Illustrative calculations were made for a simple second-order temperature and xenon poisoning power coefficient. A stability criterion was derived for the spatial flux distribution of an annular core model. Reasonably good agreement was obtained between the calculated stability margin and period of the spatial flux oscillations observed in the Shippingport Pressurized Water Reactor. Also calculated was the response of the spatial flux distribution in an annular core to both step and harmonic reactivity changes. 34 references. (auth)

C

24037 (HW-76128) PHYSICS RESEARCH QUARTERLY REPORT, OCTOBER-DECEMBER 1962. (General Electric Co. Hanford Atomic Products Operation, Richland, Wash.). Jan. 15, 1963. Contract AT(45-1)-1350. 69p.

An expression for the variational optimum kinetic response representation of reactors was obtained, and Gaussian quadrature methods integrated over a logarithmic grid were used to numerically integrate the Egelstaff S-function. The effects of small differences in the concentration of the Pu²⁴⁰ isotope in Pu-Al fuel rods on criticality were studied using a cylindrical lattice array with two zones. Critical approaches and exponential measurements were used in the experiments. Extrapolation lengths as a function of fissionable materials and calculated and measured numbers of rods required for criticality were obtained. Results indicate that the critical mass of Pu either varies more strongly with Pu²⁴⁰ concentration than the calculations show or that a systematic error is present. The P-3 computer program was used to calculate thermal fluxes in lattices with Al-Pu fuel clusters for both poisoned and unpoisoned cases. Values of η thermal and f , the thermal utilization, were calculated. Similar calculations were performed on a lattice fueled with PuO₂-UO₂. Sensitivity of the calculated flux traverse to perturbations in input parameters was also investigated. A series of PTR experiments was analyzed using a multi-energy transport group code to derive values of k_{∞} for Al-Pu fueled moderated lattices and to predict reaction rates for various resonance detectors throughout a lattice cell. Cadmium ratios at cell boundaries were derived and were found to fall short of measured values. Results of an experiment in which 13 rod clusters of both high and low exposure Al-Pu fuel were heated were analyzed to obtain values for the difference in the change in

the neutron multiplication factor as a function of temperature, and values of the effective resonance integral as a function of temperature were calculated. Measurements of the reactivity coefficients of Al-Pu fuel rods were made in the PCTR to verify the relative Pu content as given by chemical analysis and to investigate the uniformity of Pu concentration. The momentum distribution of β particles from the decay of Lu^{176m} was measured; two components with relative intensities of 44 and 56% were identified. Internal-conversion electrons from Hf^{176} L and M shells were observed for an 88.4 keV γ transition also. A ratio of the intensity of the L to M electrons of 3 ± 0.3 was obtained. A value of zero was selected for the quantum number k of the Lu^{176m} level. Critical mass measurements were continued on plutonium nitrate solutions in a 14-inch diameter sphere to determine the effects of concrete reflectors on the criticality of the vessel and the effect of an air gap between these reflectors and the core. The results verify that a 10-inch layer of concrete is a better reflector than water; the critical concentration was about 10 to 12% less than when the sphere is reflected with water. The effect of an air gap between the core and reflector varied with Pu concentration and total nitrate. Experiments with a water-reflected sphere were also performed. (D.C.W.)

28230 VARIATIONAL CALCULATIONS OF THE EXTRAPOLATION LENGTH OF SOME FUEL ELEMENTS, WITH APPLICATION TO THE THERMAL UTILIZATION FACTOR. P. Basso, B. Montagnini, and V. Pierpaoli (AGIP Nucleare, Milan). *Energia Nucl. (Milan)*, 10: 237-46 (May 1963). (In English)

The application of the results of a previous work concerning the extrapolation length of fuel and absorber elements to the calculation of the thermal utilization factor is discussed. Some calculations were performed to check the accuracy of the various approximations used. In all cases examined, the method of homogenizing the element proved to be sufficiently accurate, as far as the extrapolation length is concerned. When this quantity is determined, an elementary formula gives explicitly the thermal utilization factor (in the one-group theory). (auth)

A

28412 THE VARIATIONAL METHOD APPLIED TO THE MONOENERGETIC BOLTZMANN EQUATION. PART I. G. C. Pomraning and M. Clark, Jr. (Massachusetts Inst. of Tech., Cambridge). *Nucl. Sci. Eng.*, 16: 147-54 (June 1963).

The variational method as applied to the monoenergetic

integro-differential Boltzmann equation is investigated. It is shown that rendering the Lagrangian stationary with respect to small changes in the directional flux and adjoint directional flux is equivalent to solving the Boltzmann and adjoint Boltzmann equations. Topics discussed include the use of variational weight functions, the inclusion of boundary terms in the functional, the interpretation of a variational optimum for a nonself-adjoint operator, and the second variation. It is shown that, for the general trial function ensemble and within a special restricted trial function ensemble, the variational method is a saddle point principle. The formalism developed is applied to the angular expansion in polynomials of the directional flux. (auth)

A

28413 THE VARIATIONAL METHOD APPLIED TO THE MONOENERGETIC BOLTZMANN EQUATION. PART II. G. C. Pomraning and M. Clark, Jr. (Massachusetts Inst. of Tech., Cambridge). *Nucl. Sci. Eng.*, 16: 155-64 (June 1963).

The monoenergetic integro-differential Boltzmann equation with an arbitrary scattering kernel is transformed to a self-adjoint form and the corresponding Lagrangian written. It is shown that this transformation results in a loss of the continuity (neutron conservation) information contained by the Boltzmann equation. This information is recovered by writing the directional flux as the sum of an even and odd function (in angle) and considering a self-adjoint Lagrangian for only one portion (even or odd) of the directional flux. This procedure is shown to be equivalent to separating the nonself-adjointness from the Boltzmann operator. Further, it is shown that this self-adjoint principle is an extremum principle if the mean number of secondaries per collision is less than one. This self-adjoint formalism is applied to the angular expansion of the directional flux which results in an improved diffusion theory. Numerical results for the linear extrapolation distance and diffusion coefficient are compared with the classical $(P-1)$ diffusion theory. (auth)

A, B, C

28437 (KAPL-2000-20) REACTOR TECHNOLOGY REPORT NO. 23—PHYSICS. (Knolls Atomic Power Lab., Schenectady, N. Y.). May 1963. Contract W-31-109-eng-52. 123p.

A geometrically simple homogeneous assembly (SHA, described herein) was designed and built to provide data

such as criticality and flux distributions for comparison with such quantities obtained theoretically from few-group or multigroup treatments. Since the calculated information depends very little on geometric or other mathematical complexities, these comparisons will be highly useful in the evaluation of group constants produced from basic nuclear data. For example, the diffusion length for thermal energy neutrons can be measured or calculated from theoretical models for the scattering of thermal neutrons. Good agreement between the two methods is reported here for the cases of room-temperature water and polyethylene; indeed, the agreement is good for water over a wide range of temperatures, but fails for hot polyethylene. The reactivity coefficient of aluminum has consistently been calculated to be larger than measured. Theoretically derived modifications for the MUFT tape aluminum data still fail to produce an acceptable coefficient, but improvements in the model for reactor calculations have reduced the disagreement from 80 to 45%. The preparation of multigroup data (such as that for MUFT, above) has typically been a very laborious affair, involving many weighted averages of microscopic nuclear data. To make this a faster and more efficient process, the computer program CADAVER has been developed to perform required averaging for both elastic and inelastic scattering and smooth capture of neutrons. Further work to increase the accuracy of MUFT was performed to provide resonance self-shielding factors for aluminum and zirconium. The factors that are supplied account for decreased resonance absorption resulting from resonance scattering. The use of flux and power distributions in reactor cores results from a diffusion theory approach to neutron transport; an alternative to this is described. Employing such directly meaningful and interrelated quantities as reflection and transmission probabilities and unidirectional neutron currents in slab geometry, mathematical relations can be established for criticality and for disadvantage factors. Pile oscillator techniques have been recognized as being more sensitive than static methods for determining resonance integrals. An especially sensitive oscillator system was installed in the Thermal Test Reactor (TTR); preliminary measurements with hafnium samples indicate that an accuracy of 0.002 ϵ is available from data accumulation over a one minute period at 100 w power. This permits measurements on very minute samples or on very weakly absorbing materials. On the theoretical side, development was begun on a program, MOST, which serves to compare experimental cross-section data (microscopic) with the results of quantum mechanical calculations and to provide values of theoretical nuclear parameters which yield best fits to the measured data. A description is given of a technique whereby MOST can also be applied to the solution of rather general and complicated equations. Current best values for

fast-neutron scattering cross-sections in aluminum and zirconium, and resonance parameters in zirconium, were established through theoretical calculations and review of recent data. The new information thus developed was incorporated in MUFT data tapes. Estimates on the extent to which calculated criticality values are affected by changes in nuclear data were obtained. Reactivity changes were tabulated for sets of calculations for water-moderated systems containing either zirconium or aluminum; these changes were produced by altering elastic or inelastic scattering in the metal. Ten percent changes in large portions of the data were observed to introduce roughly a 0.5% change in k . The sixth edition of the chart of the Nuclides was issued. The detailed behavior of neutron distributions, in energy and/or spatial dependence, was examined with a number of techniques. Special distributions of thermal neutrons in hydrogen-moderated fuel-bearing assemblies were measured and calculated for a variety of mixtures and temperature conditions. Very carefully performed chopper measurements were calculated satisfactorily with a bound hydrogen kernel for systems with low absorption rates. For systems more highly loaded with fuel, a free hydrogen kernel produces best agreement with experiment; this unsatisfactory situation is not understood. Space-dependent spectra at epithermal energies can be calculated with such programs as P1MG or TET, but only at rather high cost in computer time. Work was done to learn whether the application of variational techniques to P1-type equations could reduce computer time without loss in accuracy. The MUSKRAT program, developed in these studies, has not been proven to be a satisfactory successor, although it is nearly as good as P1MG. Several lattices were examined in detail with both P1MG and TET, to determine whether the low-order angular approximation in the former produced significant errors in the epithermal spectra (and group constants). Although infinite-medium spectral calculations would be inaccurate for the lattices considered, the results with a simple P1 anisotropy proved to be sufficiently close to the double P5 available from TET results. Several formulations were developed for the calculation of approximate blackness coefficients. Ranging from a diffusion theory approach to use of integral theory variational expressions for slab transmission, reflection and absorption, the methods were relatively poor or good; the latter yielded results that differed by 2% or less from exact values for a slab of any selected optical thickness and scattering properties. Calculations and experiments are reported for the scattering cross section, and for the energy distribution of thermal neutrons in cold (-198°C) polyethylene. Agreement between the measured and calculated spectral data at this low temperature adds confidence in the theoretical model employed for the scattering. The analysis of slowing down experiments conducted with low-energy neu-

trons has long been viewed as a possible way of verifying theoretical scattering kernels. Only recently, however, have sufficiently accurate experiments been performed to bolster this view. Although the results available are not yet conclusive, those presented here show improved agreement between theory and experiment. A very accurate yet simple method of determining the absorption coefficient and reflected angular distribution for a semi-infinite plane slab with an arbitrary ratio of scattering to absorption is described. Results obtained with this method show a nonlinear variation of the emergent flux with the cosine of the angle of emergence. When this is a strong effect, even a double P1 approximation may lead to an unsatisfactory flux description. A variational method for nonlinear reactor kinetic problems is reported. (auth)

E, B

38057 A VARIATIONAL PROCEDURE FOR CALCULATING FAST GROUP CONSTANTS. P. A. Ombrellaro and F. D. Federighi (Knolls Atomic Power Lab., Schenectady, N. Y.). Nucl. Sci. Eng., 16: 343-56(Aug. 1963).

A variational procedure for calculating fast energy few group constants is described. For a given medium, the method permits one to express the flux and current solutions of the Boltzmann equation, treated according to P-1 slowing down theory in each group of a few group scheme, as a linear combination of base flux spectra to obtain the group flux and as a linear combination of base current spectra to obtain the group current. The coefficients for combining the base spectra are provided by the theory and depend only on the concentrations of the component elements of the medium. Once the flux and current spectra in each group are calculated, the group constants for the medium can be easily calculated from base flux spectra weighted library microscopic cross sections. Group constants calculated in this manner agree well with those obtained from the MUFT V program. (auth)

F

38511 A STUDY OF THE VARIATIONAL PRINCIPLES OF NUCLEAR REACTOR PHYSICS. Paul Wesley Dickson, Jr., Thesis, Raleigh, N. C., North Carolina State Coll., 1962. 203p.

The application of the two main uses of the variational principle to reactor physics is explored. The two uses are first, to derive exact equations of motion from a stationary Lagrangian and to study the implications of the Lagrangian regarding constants of the motion, and second, to obtain approximate solutions to non-linear differential equations.

Attention is restricted to diffusion theory, and hence the results are applicable only to reactors for which diffusion theory is adequate. Lagrangians for one- and two-group diffusion theory as well as one for the one-group theory including delayed neutron effects are presented and proved to be correct. From these Lagrangians the corresponding reactor Hamiltonians are found and used to determine the "conjugate momenta" and some constants of the motion for reactor diffusion theory. The implications or interpretations resultant from these constants and the conjugate momenta are discussed. The Hamilton-Jacobi equations for the one-group Hamiltonian were derived and shown to yield the expected results. Time-dependent perturbation theory is shown to be applicable to reactor problems to determine flux functions as a function of time. Variational theory is compared to the perturbation theory when carried to higher orders for the one-group case. Two-group variational theory is discussed in general terms. A modified form of variational theory which will delineate a lower bound to an eigenvalue was applied in general form to reactor problems. The standard upper bound variational theory and its lower bound form were then applied, using a one-parameter trial function, to many specific cases whose upper bounds are known from perturbation theory. Two-parameter one-group variational theory was considered. It was found that a two-parameter trial function yields quite accurate results even for heavily loaded reactors, if the poison is not too greatly concentrated. The special case of the reflected reactor was considered, and a correction to the perturbation theory determination of δk_{eff} was derived. A two-parameter variational technique was applied to this same problem and the correction required for reflected reactors is discussed. Two methods of attack, using the fourth-order differential equation and the matrix equation, on the two-group variational problem are presented. Solutions for several one- and two-parameter trial functions were determined for general poison distributions. An analytical solution for a perturbed reactor system was obtained, and the exact answer was compared with the variational answers for differential trial functions to demonstrate the validity of the method. (Dissertation Abstr.)

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F

476 (UCRL-TRANS-169) ON SOME VARIATIONAL PRINCIPLES IN THE THEORY OF OPERATIONAL EQUATIONS. M. M. Vainberg. Translated by H. P. Kramer (Univ. of California Radiation Lab., Berkeley) from *Uspekhi Mat. Nauk*, 7: No. 2, 197-200(1952). 6p.

Theorem A was derived as a special case of the Lyusternik proposition for conditional extremals of functionals, for two real functionals defined on a real Banach space E and differentiable at the point x_0 . Theorem B is derived in the real Hilbert space H for a given operator, $F(X) = \text{grad } f(x)$, and a completely continuous operator, $A(x)$, showing that the equation: $\mu x = A^2 F(x)$, has in each sphere $\|x\| \leq r$ of the space H at least two different solutions corresponding to real values of the parameter μ , where μ is a real number. Theorem C is derived as a proof for Theorem B for the case when the completely continuous operator is self-adjoint. It is noted that an application of Theorems B and C results in the existence theorems for characteristic functions for integral equations of the Hammerstein and Lichstein type. (B.O.G.)

G

743 ASYMETRIC CORE EXPERIMENTS AND THEIR ANALYSIS BY A THREE-DIMENSIONAL CODE AND TWO FLUX SYNTHESIS TECHNIQUES. R. J. Roseberry and T. F. Ruane (Knolls Atomic Power Lab., Schenectady, N. Y.). *Trans. Am. Nuclear Soc.*, 4: No. 2, 285-8 (Nov. 1961).

G, B

745 SYNTHESIS OF THREE-DIMENSIONAL FLUX SHAPES. S. Kaplan (Westinghouse Electric Corp., Pittsburgh). *Trans. Am. Nuclear Soc.*, 4: No. 2, 287-8 (Nov. 1961).

C

3385 (KAPL-M-NCF-3) NON-LINEAR EQUATION VARIATIONAL METHOD. N. C. Francis (Knolls Atomic Power Lab., N. Mex.). July 10, 1961. Contract W-31-109-Eng-52. 6p.

A nonlinear variational method is presented for the inhomogeneous case of power reactor physics problems. (D.L.C.)

F

9575 (HW-68284) THE VARIATIONAL METHOD AND REACTOR PHYSICS. C. W. Lindenmeier (General Electric Co. Hanford Atomic Products Operation, Richland, Wash.). Jan. 26, 1961. Contract [AT(45-1)-1350]. 18p.

Some of the less familiar mathematical methods required in developing a variational formalism for reactors are described and applied to obtain the multi-group approximation. Linear spaces, adjoint spaces, linear operator, adjoint operators, Green's function, physical interpretation of the adjoint, variational methods, variational functional in the diffusion, trial functions, the multi-group approximation, and overlapping and non-overlapping trial spectra are discussed. (M.C.G.)

F, C

9653 (KAPL-2000-16) REACTOR TECHNOLOGY REPORT NO. 19—PHYSICS. (Knolls Atomic Power Lab., Schenectady, N. Y.). Dec. 1961. Contract W-31-109-Eng-52. 192p.

Nuclear Cross Sections. The resonance integrals of Mn, Hf, and Nb were measured. The mass spectrometer filament enrichment technique was used to determine the Sm^{149} effective activation cross section. The $\text{O}^{16}(n,p)$ cross section was measured as a function of neutron energy from 12.5 to 16.5 Mev. The $\text{F}^{19}(p,\alpha\gamma)$ reaction was used to calibrate a 1-Mev Cockcroft-Walton accelerator used to accelerate deuterons for producing neutrons. ABACUS, a computer program for cross-section calculations, is described. ABACUS was used to calculate the optical model parameters of the angular distributions of neutrons scattered from Zr^{90} at 0.25 to 7 Mev. Methods are described for calculating elastic and inelastic differential cross sections, angular momentum coupling coefficients, and compound nucleus effects in neutron capture. Results are given for the development of a satisfactory potential function in the direct interaction model. Photoneutron production calculations for Be^9 and C^{13} were extended to an energy range $E_\gamma \approx 3$ Mev. Computational work on the cross section of the thermal neutron scattering by water and polyethylene is described. Neutron Spectra and Fluxes. The thermal neutron scattering kernels were used in calculating neutron spectra in both infinite homogeneous media and lattices. Computer programs for the calculation of neutron spectra are described, e.g., TET, TRANSWAKRUM, and TRAM. The application of variational techniques to linear systems is discussed. Few-Group Parameters: Criticality. A

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two-mode variational procedure was developed for calculating fast-group diffusion theory parameters. In the variational calculation of thermal spectra, it is shown that the advantages of symmetrization of the scattering kernel are nil. Methods are given for computation of spectrum-averaged cross sections. The temperature dependence of the thermal diffusion length in paraffin was measured and used to derive transport cross sections. The cross sections for fission products were examined in detail with respect to their poisoning effect on reactor cores, and it is concluded that, while uncertainties in current data give small errors, the assumption that stable products are formed directly in the fission process can lead to significant errors. Variation calculation of the reactivity k is discussed. Reactor Kinetics. The method for calculating the effect of neutron population fluctuations on reactor design was extended to account for the actual nonzero lifetime of prompt neutrons. A semi-direct variational method is given for solving the space-time neutron multi-group kinetic equations for reactor analysis. (D.L.C.)

G, B

12413 MULTICHANNEL FLUX SYNTHESIS. E. L. Wachspress, R. D. Burgess, and S. Baron (Knolls Atomic Power Lab., Schenectady, N. Y.). Nuclear Sci. and Eng., 12: 381-9(Mar. 1962).

A procedure is described for calculating neutron fluxes at nodes of a three dimensional grid with few points, in planes perpendicular to a selected axis. Difference equations relating fluxes within each plane are determined in advance from detailed two-dimensional studies. This "multichannel synthesis" model may be used for parameter studies and dynamic analysis. Comparison of a representative calculation with results of a detailed three-dimensional computation is given. (auth)

F

14254 A VARIATIONAL PRINCIPLE FOR NONLINEAR SYSTEMS. Jeffery Lewins (Staff Coll., Camberley, Surrey, Eng.). Nuclear Sci. and Eng., 12: 10-14(Jan. 1962).

The equations describing a reactor system are sometimes nonlinear and do not admit a solution for the neutron density that is separable into a function of time only and a function of the remaining variables. An appropriate variational principle is given by demanding that the calculation of the observable nature of the reactor is insensitive to the

value employed for the density, thus obtaining an equation for the optimum distribution of detectors to measure the observable behavior. This optimum weighting function is not identical with the conventional adjoint function or importance in the nonlinear range but the conventional treatment of linear systems is found to be a special case of the general principle. It is shown that the approximate treatment of nonlinear systems as eigenvalue systems is fundamentally unsound. (auth)

E, B

18409 DIFFUSION APPROXIMATION AND VARIATIONAL EXPRESSION FOR THE THERMAL NEUTRON DISTRIBUTION IN SPACE AND ENERGY. Akinao Shimizu (Nippon Atomic Industry Group Co. Ltd., [Japan]). Nippon Genshiryoku Gakkaishi, 4: 161-6(Mar. 1962). (In English)

The diffusion approximation for the motion of neutrons which move in the energy space as well as in the ordinary space is derived from the diffusion approximation for the motion of mono-energetic neutrons. Under this approximation, the variational expression for the space-energy distribution of thermal neutrons is derived. The functional in this expression becomes minimum for the actual distribution. The derived variational method is applied to the problem of the thermal neutron spectrum in a heterogeneous lattice. (auth)

A

19733 AN EXPRESSION FOR THE NEUTRON BLACKNESS OF A FUEL ROD AFTER LONG IRRADIATION. Hisao Yamakoshi (Transportation Technical Research Inst., [Japan]). Nippon Genshiryoku Gakkaishi, 4: 244-50(Apr. 1962). (In Japanese)

A variational expression is derived for the neutron blackness β of a fuel rod which has inhomogeneous distribution of nuclear elements depending on the distance from its axis. The following assumptions are made: scattering within the rod is isotropic in the coordinate system and does not change the energy of an entering neutron; the incident neutron current is uniform on the surface of the rod and is isotropic in solid angle; and monochromatic treatment is allowed. The evaluation of the variational expression for β by using the solution of transport equation for neutrons in the fuel rod as a trial function shows that when a lot of Pu^{239} accumulates in the vicinity of the rod surface and U^{235} remains near the axis of the rod, the blackness β in the energy 0.3 eV is increased much more due to the scattering than in the case of the homogenized rod. The values of β for the homogenized rod are compared. Both results agree fairly well as the ratio of absorption to scattering cross section increases. (auth)

G

19735 SOME NEW METHODS OF FLUX SYNTHESIS. S. Kaplan (Westinghouse Electric Corp., Pittsburgh). Nuclear Sci. and Eng., 13: 22-31 (May 1962). (WAPD-T-1374)

A variational principle is applied to develop a method for constructing approximate solutions to the three dimensional, few group diffusion equations using only the solutions of one and two dimensional problems. Three additional related methods are described and a numerical example presented which is representative of several that were studied. It is found that the new methods give considerably improved results compared with the conventional synthesis method. (auth)

C

22975 THE SPACE-TIME NEUTRON KINETIC EQUATIONS OBTAINED BY THE SEMIDIRECT VARIATIONAL METHOD. D. E. Dougherty (Knolls Atomic Power Lab., Schenectady, N. Y.) and C. N. Shen. Nuclear Sci. and Eng., 13: 141-8 (June 1962).

Using a semidirect variational method, the time-dependent coefficients of a modal expansion of the neutron fluxes are given by the Euler-Lagrange equations obtained from a variational principle for the multigroup kinetic equations. In order to avoid the difficult task of determining orthogonal eigenfunctions for a modal expansion in a complex geometry, an approximate solution of the kinetic equations by the method of the Green's function results in a set of readily calculated space modes. These modes can also be adapted to perturbations in the diffusion parameters about which a priori information is available. (auth)

A

24362 VARIATIONAL MONTE CARLO CALCULATIONS OF FLUX DEPRESSION IN FOILS. F. R. Nakache, H. Goldstein, and M. H. Kalos (Columbia Univ., New York and United Nuclear Corp., White Plains, N. Y.). Trans. Am. Nuclear Soc., 5: No. 1, 32-3 (June 1962).

B, E

24368 A COMPARISON OF A SELF-ADJOINT VARIATIONAL METHOD TO 36-GROUP THERMAL SPECTRUM CALCULATIONS OF HETEROGENEOUS ARRAYS. S. L. Shufler (Westinghouse Electric Corp., Pittsburgh). Trans. Am. Nuclear Soc., 5: No. 1, 37-9 (June 1962).

F

24372 GENERALIZED VARIATIONAL METHOD. M. D. Kostin and H. Brooks (Harvard Univ., Cambridge, Mass.). Trans. Am. Nuclear Soc., 5: No. 1, 41 (June 1962).

B, E

24400 A VARIATIONAL PROCEDURE FOR CALCULATING FAST GROUP DIFFUSION THEORY PARAMETERS. P. A. Ombrellaro and F. D. Federighi (Knolls Atomic Power Lab., Schenectady, N. Y.). Trans. Am. Nuclear Soc., 5: No. 1, 63-4 (June 1962).

C

24543 SYNTHESIS APPROXIMATIONS IN THE TIME DIRECTION. J. A. Bewick, A. F. Henry, and S. Kaplan (Westinghouse Electric Corp., Pittsburgh). Trans. Am. Nuclear Soc., 5: No. 1, 177-8 (June 1962).

B, C

24943 (MND-C-2500-3) APWRC-SYNFAR-02, A P1 AND DSN THEORY. FORTRAN-II CODE FOR STATIC AND DYNAMIC SYNTHESIS OF TWO-DIMENSIONAL FLUX AND REACTIVITY. T. M. Olsen (Martin Co. Nuclear Div., Baltimore). May 1962. Contract [AT(30-1)-2431]. 77p.

A computer code, APWRC-SYNFAR-02, was programmed in FORTRAN II for synthesis computation of static flux and reactivity, or of stable period and corresponding flux shape in XY or RZ geometry on the IBM 7090. It also allows direct computation of the same quantities in one-dimensional spherical geometry. Running time is 12 minutes or less. (M.C.G.)

D

24944 (MND-C-2500-4) APWRC-SYBURN, A FORTRAN-II PROGRAM FOR SYNTHESIZED TWO-DIMENSIONAL P1 OR DSN BURNUP CALCULATIONS. E. A. Colbeth and T. M. Olsen (Martin Co. Nuclear Div., Baltimore). June 1962. Contract AT(30-1)-2431. 62p.

The reactor physics neutron code, APWRC-SYBURN, was designed for the IBM 7090 and written in FORTRAN-II. It was developed for one-dimensional regionwise or intervalwise determination of isotope concentrations during reactor burnup, including effects of rod or other control eigenvalue variation, providing core averaged radial constants for subsequent synthesized axial burnup problem. Typical running time is 2 to 7 minutes. (M.C.G.)

A

26540 THE REACTIVITY OF A CENTRAL AIR GAP IN A BARE REACTOR. H. Haeggblom (Aktiebolaget Atomenergi, Sweden). p.85-94 of "Physics of Fast and Intermediate Reactors. Vol. II." Vienna, International Atomic Energy Agency, 1962. (In English)

Two methods are presented for calculation of the reactivity equivalence of a central gap in a bare reactor with rectangular cross section. The first is a perturbation theory; the second is an application of variation calculus. The eigenvalue is solved for the transport equation in integral form. The principal difference between the two methods is in the assumption for the flux shape. In the perturbation theory, it is assumed that the flux is unperturbed and has the form $A \cos \pi x/L$. The reactivity as a function of the gap width is expressed in closed form in one and two energy groups, but the expression is valid only for reactivity changes less than about 8%. The variation calculus is developed only for one energy group, but the accuracy is approximately independent of the gap width. This is obtained by varying the expression for the flux in order to optimize the functional integral. The results are compared with those obtained by Chernick and Kaplan by diffusion theoretical methods. (auth)

E, B

28416 (WAPD-BT-25(1-24)) THE DESCRIPTION OF THE THERMAL NEUTRON SPATIALLY DEPENDENT SPECTRUM BY MEANS OF VARIATIONAL PRINCIPLES. A. J. Buslik (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh).

The application of the calculus of variations to the determination of the thermal neutron space and energy distribution is described, and it is shown how thin absorbing regions treated by blackness theory can be included in the method. The general method is then applied to the problem of an absorber adjacent to a large fuel bearing region. Results are compared with the 36 multigroup solution of the thermal space energy problem by the digital computer code SLOP-1. In addition, a manner of incorporating these results into a one-group design method is presented. (auth)

E, B

29713 (TID-12780) VARIATIONAL METHODS IN NEUTRON THERMALIZATION. D. S. Solengut (Knolls Atomic Power Lab., Schenectady, N. Y.). May 1961. Contract [W-31-109-Eng-52]. 9p.

Variational methods for calculating thermal flux distribu-

tion in reactors for design purposes are described. The methods relate the nature and essential features of the scattering kernel to the characteristics of the neutron distributions that have engineering significance. Information is given on variational principles, adjoint functions, and their various interpretations. Typical problems in neutron thermalization are evaluated by this method. The first problem considered is the asymptotic behavior of the neutron distribution, due to a discontinuity in an infinite medium which leads to an exponentially decaying space distribution, or a pulse in a block of material which leads to an analogous eigenvalue problem for the calculation of the decay constant for the time-dependent distribution. The second problem is the calculation of the spectrum of neutrons in an infinite medium generated by a uniform source of fission neutrons, and its use in calculating thermal group constants for simplified models of a reactor. The final problem concerns the calculation of the thermal flux distribution in space and energy in a system with many regions having different compositions and temperatures, which is necessary for accurate reactor design. (N.W.R.)

E, B

30028 (WAPD-T-1450) A COMPARISON OF A SELF-ADJOINT VARIATIONAL METHOD TO 36-GROUP THERMAL SPECTRUM CALCULATIONS OF HETEROGENEOUS SYSTEMS. S. L. Shufler (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh). Apr. 1962. Contract AT(11-1)-GEN-14. 35p.

A self-adjoint variational principle related to the thermal neutron spatial spectrum problem by Buslik was tested against a 36-group thermal code and a few-group code for typical one-dimensional heterogeneous arrays. A description of the theoretical method is given including some observations of the differences between the self-adjoint method of Buslik and the nonself-adjoint formalism reported by Calame and Federighi. A comparison of nuclear parameters, activation shapes, and local region capture fractions are presented for three types of one-dimensional arrays: absorber-fuel cells, absorber-nonfuel-fuel cells, and absorber-fuel-nonfuel cells. Some comparisons of the mixed number density method to the variational model are also included. Results indicated the utility and proof of the success of the self-adjoint variational principle in predicting the important nuclear parameters of a wide range of fuel-nonfuel-absorber combinations at both cold and hot temperatures. (M.C.G.)

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30029 (WAPD-T-1463) SYNTHESIS APPROXIMATIONS IN THE TIME DIRECTIONS. J. A. Bewick, A. F. Henry, and S. Kaplan (Westinghouse Electric Corp. Bettis Atomic Power Lab., Pittsburgh). Apr. 1962. Contract AT(11-1)-Gen-14. 19p.

Application of an approximation procedure in which the solution is approximated by a linear combination of functions which are known, to reactor problems involving the time variable is described. The approximate solution has the form $\phi(x,y,z,t) = T_1(t) \psi_1(x,y,z) + \dots + T_n(t) \psi_n(x,y,z)$ where $\psi(x,y,z)$ are spatial shapes appropriate to various ranges of t . Illustrations are given corresponding to three different time scales. (J.R.D.)

F

31273 (NP-11754(Vol.II)(Sect.IV)) SOME REMARKS ON THE USE OF VARIATIONAL METHODS IN REACTOR EXPERIMENTATION. Helge Christensen (Christian Michelsens Institutt for Videnskap og Andafrihet, Bergen). Sect. IV of ADVANCED COURSE ON IN-CORE INSTRUMENTATION FOR WATER COOLED REACTORS, ORGANIZED BY THE NETHERLANDS'-NORWEGIAN REACTOR SCHOOL AT INSTITUTT FOR ATOMENERGI, KJELLER, NORWAY, 21st AUGUST - 1st SEPTEMBER 1961. VOLUME II. 47p.

It is noted that transfer functions are used to relate input and output in systems such as reactors. The basic ideas behind transfer function techniques are reviewed and the importance of transient conditions in reactor experimentation is pointed out. The theoretical foundation for the various experimental techniques is presented. Discussions of experimental procedures are included. (J.R.D.)

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